



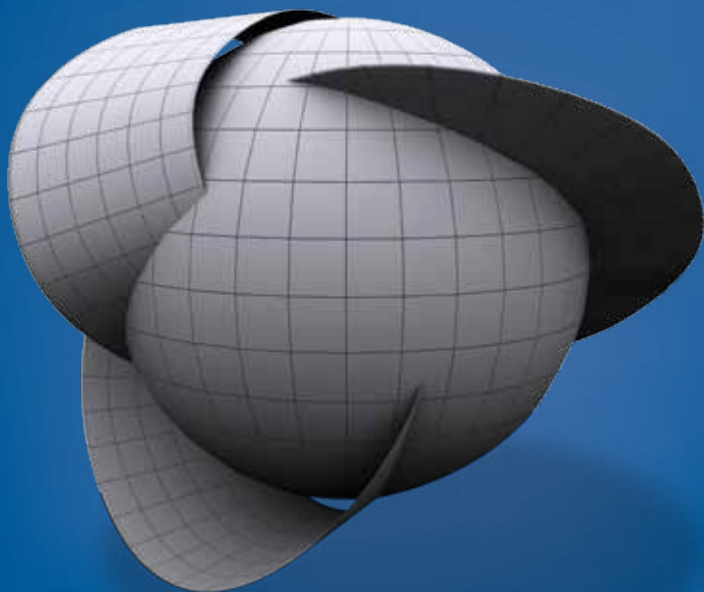
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# Superstring Theory: The DNA of Reality

Course Guidebook

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University of Maryland at College Park



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## **S. James Gates, Jr., Ph.D.**

John S. Toll Professor of Physics, University of Maryland at College Park

S. James Gates completed his undergraduate education and received two B.Sc. degrees (in mathematics and physics) at the Massachusetts Institute of Technology. His Ph.D. (in physics) was conferred for studies of elementary particle physics and quantum field theory. His Ph.D. thesis on *supersymmetry* was the first devoted to this subject at MIT. Dr. Gates's postgraduate studies started as a Junior Fellow of the Harvard Society of Fellows and ended with an appointment at Caltech. Faculty appointments began at MIT and later continued at the University of Maryland at College Park (1984–present). From 1991–1993, Dr. Gates served as physics professor and departmental chair at Howard University. In July 1998, he was named the first John S. Toll Professor of Physics, becoming the first African-American to hold an endowed chair in physics at a major research university in the United States. The Washington Academy of Sciences named him its 1999 College Science Teacher of the Year.

Professor Gates has authored or co-authored more than 180 research papers published in scientific journals, and one book, and contributed numerous articles to several books. His research, in the areas of the mathematical and theoretical physics of supersymmetric particles, fields, and strings, covers such topics as the physics of quarks, leptons, gravity, superstrings and heterotic strings, and unified field theories of the type first envisioned by Albert Einstein. Dr. Gates travels widely, speaking at national and international scientific meetings.

A member of the American Physical Society (APS), Sigma Xi, and the National Society of Black Physicists, Dr. Gates is also a past president of the NSBP and has served on the executive board of APS. He has served as a consultant for the National Science Foundation, Department of Energy, Department of Defense, Educational Testing Service, and Time-Life Books. He was the first recipient of the APS Bouchet Award and is a Fellow of the APS and NSBP; in 1997, he received the Martin L. King, Jr. Leadership Award from MIT. He is also a member of the 62<sup>nd</sup> College of Distinguished Lecturers of Sigma Xi and the board of directors of the Quality Education for Minorities Network (QEM).

The work of Dr. Gates and others was highlighted on “The Path of Most Resistance,” in the PBS series *Breakthrough: The Changing Face of*

*Science in America*. Dr. Gates has appeared in three additional PBS science documentaries, including *Einstein's Big Idea*. In March 1998, he appeared on the simultaneous C-Span television broadcast and Internet cybercast of the second Millennium Lecture by Professor Stephen Hawking from the East Room of the White House. Professor Gates was asked to provide comments on the topic of supersymmetry for broadcast and live audiences, including U.S. President William J. Clinton.

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# Superstring Theory: The DNA of Reality

## Scope:

This course aims to provide a non-technical and accessible description of the central foundational concepts and historical development of the topic in theoretical physics called *superstring/M-theory*. These lectures place this topic in the context of the more general development of mathematical and scientific thought that can be traced from the ancient realms of Egypt and Greece to medieval Iraq, Renaissance Europe, and the present.

By the end of the course, students will gain insight not only into the strange new world of superstring/M-theory but also into the central role of mathematics as the empowering element of human creativity driving the conception of science through theoretical physics. Although mathematics plays a central role in this story, it is kept to a minimum in the lectures. This is possible because of an almost unique capability of the courses produced by The Teaching Company.

For years, I have been asked to write a book covering this topic, largely because I have made more than 100 non-technical presentations on superstring theory since 1988. But no ordinary book would be capable of conveying to most people the mathematical ideas that provide the foundation of this topic. Unless and until it receives observational support, superstring/M-theory will be all about mathematics, but mathematics is largely inaccessible without highly specialized training. The experience of lecturing to nonscientists on this topic suggested to me that visual media, both still and animated, provide the key to solving this problem. The video format of Teaching Company courses is an exquisite platform for using computer-generated imagery to augment conventional lectures and books. Essentially, in these lectures, I use computers to “play” mathematics in much the same way that a musician plays scores.

The course begins with a description of this approach and a cursory look at the concept of the string. We’ll explore the strange realization that understanding the universe at its largest scales requires knowledge of the smallest structures and their behavior, together with mathematics. We’ll also look at the role of human creativity in the conception of science, and we’ll discuss a number of not-so-well-known properties of mathematics as a tool for science.

Superstring/M-theory is sometimes presented as a radical break with all preexisting scientific thought. To counter this notion, we'll pay some attention to the known structure and rules of the universe at the very smallest scales. This discussion will establish many concepts that reappear in superstring/M-theory, in particular, the concept of the quantum world.

For many years, physicists largely ignored the fact that their accepted descriptions of the largest structures and the smallest structures in the universe were incompatible. We'll see how Stephen Hawking used black holes to force a crisis in theoretical physics. The only known way out of this crisis begins with *bosonic string theory*.

We'll also review Einstein's theory of special relativity and its role in string theory, noting the presence of time as the fourth dimension and the largely overlooked role of a structure that can be called *Einstein's Hypotenuse*. The dual requirements of quantum theory and the theory of special relativity in bosonic string theory lead to a description of a *tachyon*—a particle capable of destroying the known laws of physics.

Next, we turn to two little-known properties of the electron in the quantum world that, in effect, banish the tachyon and create a new generation of spinning strings and superstrings. We return briefly to the real world for a look at electricity and magnetism in preparation for a leap in the development of this subject to the *heterotic string*, conceived in 1984. This is the first mathematical construct that realized Einstein's long-sought dream of a *unified field theory*.

We next turn to the widely discussed possibility of hidden dimensions and the little-discussed alternative, followed by an exploration of the manner in which second-generation string theories describe all forces, including gravity. We'll pay some attention to the modifications implied by the newer strings, noting the rigorous mathematical and logical support that has been given to the conjecture of Stephen Hawking. We'll also discuss new forms of energy and matter, called *superpartners*, and learn about the superpartners for the particle zoo of the quantum world, including the superpartner for Einstein's graviton, which leads to *supergravity*.

Toward the end of the course, we'll look at current attempts to use concepts from string theory to gain increased understanding of the forces and structures of matter inside the proton and neutron. We'll explore a little-discussed "hidden dimension," radically different from all others, and its role in the concept of *superspace*. We'll close with the ultimate

*supergravity theory*, associated with a world of eleven hidden dimensions, and its connection to the mother theory of strings, M-theory. As the lectures come to an end, we'll look toward the possibility of elucidating unsolved problems and meeting the challenges of this class of mathematical and scientific ideas.

# Lecture One

## The Macro/Micro/Mathematical Connection

**Scope:** Scientists for several thousand years have been trying to understand the universe. They have reached a stage at which numbers of features are understood, yet much more is not. One of the places this quest has led is to an idea called *string theory*. String theory is not really physics yet. It's mathematical physics; it's theoretical physics, but its realization has not been seen in the laboratory. This lecture begins with some “real” physics, then, using this as a background, moves on to string theory.

### Outline

- I. Our universe contains *binary stars*—two stars that orbit one about the other—and sometimes, these stars undergo strange transformations to become *neutron stars*.
  - A. Neutron stars have strong magnetic fields about them and emit high-energy particles called *gamma rays*. Our animation shows a pair of neutron stars in space suspended in a “substance” that might be mistaken for water. An imagined surfer on this substance rides as it travels through space.
  - B. The substance distorts the light of stars, and a wave of the substance might encounter the Earth.
    1. What will be the impact of this wave on Earth?
    2. The propagation of this wave might lead to two large L-shaped buildings in the United States. These are called LIGO, the Laser Interferometer Gravitational-Wave Observatory.
    3. The wave causing the buildings to move (in the animation) seems like science fiction, yet the U.S. has built these two facilities at a cost of about \$400 million to detect the existence of these waves.
    4. Where did the idea about the existence of the waves come from? The answer lies in the realm of theoretical physics.
- II. The idea of string theory is that the smallest, most fundamental objects in our universe are not similar to little balls, as has been thought for

about 2000 years, but instead, might be more similar to filaments of spaghetti.

- A. An animation helps visualize rapidly shrinking in size by factors of 10.
- B. At  $10^{-10}$ , entry to the realm of the atom occurs; at  $10^{-15}$ , we encounter the realm of the nucleus, with its protons and neutrons. With continued shrinking, we catch a glimpse of quarks presently thought to be permanently bound in the interior of nuclear matter.
- C. With more shrinking, realms that can be probed by human instrumentation are left behind. Now the journey truly enters the unknown—the region where scientific knowledge of our universe ends.
  - 1. For about 30 years, there has been mounting mathematical evidence that the universe, at the level of  $10^{-34}$  or  $10^{-35}$  meters, is described by ultramicroscopic filaments, which may well be the DNA of our universe.
  - 2. This is the realm of string theory, and these lectures are designed to give unprecedented insights into what physicists have been able to glean from the mathematical evidence about strings.

**III.** Why would scientists want such a strange picture of the universe? To answer this question, we must consider attributes of the universe that are fairly well established.

- A. Apparently our universe was spawned about 14 billion years ago in an event called the *Big Bang*.
  - 1. In 1916, Albert Einstein wrote a masterful equation creating the *Theory of General Relativity*. One of the predictions of this theory was that, at some point in the past, a universe burst into existence and continued to expand forever after at a constant speed.
  - 2. Einstein had mastered, figuratively, what might be called in science the *fudge factor*. Initially, he wanted his equations to give a picture of a static and eternal universe.
  - 3. Instead, his equations initially described a universe that exploded into existence, then grew. By introducing the fudge factor, now called the *cosmological constant*, he was able to mathematically describe a fixed bubble—the universe.

- B. If string theory is to be a successful theory of nature, it must describe all of the universe's attributes, such as *dark energy*, believed to be responsible for the present accelerating expansion of the universe.
  - C. String theory must also explain something even more mysterious called *dark matter*, which seems to be what allows galaxies similar to our own to be stable over the length of time that represents its age.
- IV. String theory really is about science, but one might ask: How can science probe realms not accessible by current instrumentation or human senses?
- A. Science is a man-made construct, consisting of three parts: objective reality, the human imagination, and technology.
  - B. It is with the use of mathematics that theoretical physicists probe the structure of the universe in the hope of not working solely in the region of the imagination but also in the realm of objective reality.
- V. How does mathematics come into this story?
- A. Scientists take measurements and the accumulated data can be plotted in various ways. Theoretical physicists try to understand how these kinds of plots are represented in terms of mathematical equations.
  - B. Mathematics implies knowledge of the universe about which humanity has no other *proven* way of finding.
    1. For example, the Big Bang was followed by a period when the densities were so high that light could not propagate.
    2. The universe was very small, but at a certain point, it spread out enough that light (photons) could travel throughout it.
    3. When this happened, a first dawn occurred—the first morning of existence. The light left behind what physicists call the *surface of last scattering* and went streaming through the universe.
    4. In the 1960s, some of these photons, in the form of radio waves, were detected by Penzias and Wilson, operating a device called the Big Ear. As they listened to the universe, they heard a kind of static but initially did not understand its source.

5. At Princeton, however, other scientists knew if a Big Bang had occurred, there must be left afterward a kind of sizzling that could be detected everywhere in the universe. This is now called the *Cosmic microwave background (CMB)*; it offers a snapshot of the universe when it was about 300,000 years old.
6. This is a remarkable story considering that the idea of the Big Bang all started as mathematics.

**VI.** The simplest picture of a string is of a very tiny piece of “spaghetti” that, with a bit of tension, might support a type of wave. We would expect this string to behave like strings in our world, that is, it should be able to “sing.”

- A. When strings were first written as equations, physicists realized that two types could exist: With an *open string*, the ends are free to flop around; with a *closed string*, the two ends are connected.
- B. In these lectures, we will work toward understanding why (and to a lesser degree how) physicists have come to these strange pictures about how the universe is put together and how it fits with real science.
- C. It cannot be emphasized too strongly that, at this point in time, string theory is mathematics.
  1. In these lectures, the use of a computer as a device to “play” mathematics (in the same way that instruments play music) will be critical.
  2. Whenever a physicist writes an equation, in the back of his or her mind is a picture of some situation. The waves shown near the beginning of the lecture are pictures described by mathematics.
  3. Physicists build these pictures in the imagination by writing equations that support the way these pictures work and evolve.

**VII.** These pieces of “spaghetti” have other attributes, which will be important for future lectures, and one of these attributes is *spin*.

- A. In a science class, a teacher might explain that an atom consists of protons, neutrons, and electrons.
  1. Picture an electron as an infinitely small ball. In the interior of protons and neutrons, visualize smaller pieces of matter,

called *quarks*. Both quarks and electrons behave like tiny spinning balls.

2. In the 1930s, science discovered that objects this small almost always seem to spin at the same rate, either clockwise or counterclockwise, without being able to speed up or slow down.
  3. Is it possible to incorporate this type of spinning property into these pieces of spaghetti? The answer is yes, and it leads to a second generation of strings, called *spinning strings*, or *superstrings*.
  4. Not only does the spaghetti vibrate and jiggle, it also, in some sense, spins on its axis.
- B.** Another look at one of the most interesting and well-received attributes of string theory, little known to non-string scientists, is in order.
1. Imagine holding a closed string in one's hand as it vibrates. A very careful examination might reveal that not all of the string is actually moving. Some places, which physicists call *nodes*, are at rest.
  2. Before 1984, all physicists in the world thought that closed strings had to have this behavior. But once recognition of this behavior exists, it is possible also to imagine other possibilities.
  3. In 1984, four physicists at Princeton University imagined a different kind of string, in which the nodes were in motion. If it is possible to write the mathematics describing this, it then becomes possible to describe a new kind of string.
  4. Further, if the nodes are rotating in a clockwise sense, should we be able to discover the mathematics that has the nodes rotating in a counterclockwise sense? The answer is yes.
  5. The four gentlemen at Princeton, now called the Princeton String Quartet, first wrote the equations that described all these motions; the result was *heterotic string theory*.
  6. Einstein spent the last 30 years of his life in a frustrating search for a *unified field theory*, that is, mathematics that could describe gravity, electric and magnetic forces, the strong and weak nuclear forces (to be discussed in later lectures), and particles, such as electrons. When the equations of heterotic

string theory were written, it became clear that Einstein's dream had been realized.

7. Einstein failed in his search, and by the time of his death, most physicists thought this quest was a pointless one. The equations of the Princeton String Quartet proved that Einstein's idea was a vital piece of science. A unified field theory in the form of heterotic string theory was possible.
- C. Other interesting attributes of strings that make them seem even more like science fiction will be covered in these lectures; one of these is the notion of extra dimensions.
1. Many people have come across the notion that there may be hidden dimensions of our universe; this idea may or may not be true.
  2. The mathematics of string theory is so complicated that, although most physicists believe in these extra hidden dimensions, your lecturer and his collaborators, as well as several other groups of physicists in the late 1980s, studied the equations and found there are versions of string theory in which there are no hidden dimensions. Which is right, in the sense of more accurately describing our universe?
  3. For string theory to become a true piece of science, it is necessary to obtain signals from nature that imply, in some way, that this one set of mathematics describes the way the universe works.
- D. In these 24 lectures, there will be much attention paid to mathematics, on one hand, and science (i.e., physics), on the other hand, in an attempt to knit these two together in a coherent picture to give the student more insight into what is going on at this very strange boundary called superstring/M-theory.

### Readings:

Bartusiak, *Einstein's Unfinished Symphony*.

Cheetham, *Universe: A Journey from Earth to the Edge of the Cosmos*.

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Hawking, *The Universe in a Nutshell* and *A Brief History of Time*.

Kaku, *Hyperspace*.

Motz and Weaver. *The Story of Physics* and *The Story of Mathematics*.  
Tyson and Goldsmith, *Origins: Fourteen Billion Years of Cosmic Evolution*.  
Weinberg, *The First Three Minutes*.

**Questions to Consider:**

1. What part does mathematics play in string theory?
2. What quest of Einstein's does string theory help fulfill?

## Lecture Two

### Who Is Afraid of Music?

**Scope:** Of all the languages humanity uses, only mathematics has shown the capability to describe nature’s most fundamental laws and structures. Why is this so important in a series of lectures about string theory? The evidence for strings—for the belief that, at a fundamental level, our world is “defined” by pasta-like structures—can be found presently only in mathematical equations. Mathematics has many interesting and even spooky properties, but above all, it allows humans to understand aspects of our world that have no other way of being comprehended.

### Outline

- I. Your lecturer has sometimes been described as a “heretic in the church of string theory.” Why? Perhaps due to his belief that it is not necessary to be a string theorist or a physicist to “understand” string theory at more than simply a cursory level.
  - A. This course will allow students access to the pictures and models that describe string theory, although it will, necessarily, be less precise than the understanding required for physics research.
  - B. Though the subject is considered to be complex and incredibly difficult, a student should not become frustrated if it is necessary to watch these lectures more than once. Even some physicists feel they understand portions of string theory less than other subjects in physics.
  - C. Science is not a linear process, and the progress in this course will reflect the way science really works. It will take a few lectures to get to string theory since science builds on what has come before. We will need to talk first about the theories of Newton, Maxwell, and Einstein, as well as dark matter, dark energy, antimatter, and black holes. The course will take up some concepts, such as particle physics, and return to them time and time again.
  - D. Science—especially physics—is not about eternal truths. The essence of science is the notion of falsification—showing through experiments that a concept, proposal, or idea is wrong. Even when string theorists seem to have thrown out all vestiges of reality,

there is still a commitment to the notion that this work can ultimately be tested. The commitment to the idea that a belief can be shown to be false by the efforts of humans is one of the most profound distinctions between science and other human belief systems. Every new generation of scientists is charged with checking the entire scientific canon that it inherits in the attempt to prove it false.

- E. Throughout these lectures, the examination of string theory through mathematics—through the equations—will lead students to the conclusion that string theory represents the only theory presently capable of describing our world and all the essentials of our existence.

**II.** Mathematics is a language like other languages, but it has some differences.

- A. Mathematics can be described as an organ of perception, an extra-sensory perception organ—ESP. Theoretical physicists (more generally, scientists) use mathematics to gain an insight into physical reality that no other means can provide.
- B. An example of this is an atom. Atoms are commonly discussed, yet it wasn't until 1905 that Einstein was able to predict the size of atoms, using mathematics and his observations of the world.
- C. The precision with which mathematics can transmit ideas and concepts is striking when compared to other forms of human communication.
- D. Mathematics has another property, called *unreasonable effectiveness*, which even theoretical physicists find surprising. Physicist Eugene Wigner noted that mathematics has enabled accomplishments in science over the last few centuries far beyond what might have been expected.
- E. Another example of the “spookiness” of mathematics can be found in the famous story of Sir Isaac Newton and the apple. Newton's equation describing why the apple fell on his head also happens to be the same equation that describes why the Moon stays in orbit around the Earth. Through the mathematics of the description of gravity, science discovered that these are two different aspects of one phenomenon.

- F.** In the 1930s, a physicist named Paul Dirac attempted to write an equation to describe an electron in a more accurate manner. Up until that time, scientists knew how to describe electrons consistent with the equations of Newton but not with those of Einstein. Scientists also knew how to describe electrons using the laws of quantum theory. Both quantum theory and Newton’s laws were known, but nobody knew how to describe an electron that obeyed Newton’s laws, quantum theory, and simultaneously, Einstein’s equations.
- 1.** Dirac set himself the goal of solving this problem, which he did. He wrote what is now called a *relativistic equation* for the electron that is, indeed, consistent with the laws of very small objects and the laws of special relativity as given to us by Einstein.
  - 2.** Dirac also found a second solution to his equation that described an object that had all the properties of an electron but the opposite charge. More remarkably, when this second mathematical solution is combined with the first, they annihilate each other and leave behind a puff of energy, exactly obeying Einstein’s  $E = mc^2$ .
  - 3.** Dirac had discovered what is now called *antimatter*, a mirror image of the matter in everyday experience. Before Dirac wrote his mathematics, no one had ever dreamed of such a thing.

**III.** It is useful to compare the reaction of most people to pages of equations versus pages of a musical score.

- A.** As a scientist who uses mathematics, this lecturer finds it interesting to note that a musical score is about as complicated as a page of equations, yet the reaction that most people have to music is diametrically opposed to the reaction they have to mathematics.
- B.** When musicians look at a score, they can access an experience that is out of reach for many who are not trained as musicians. This is so because they are able to read musical scores. Theoretical physicists and scientists also compose scores, but not musical scores—mathematical scores.
- C.** Further, theoretical physicists can “hear” the music that’s behind a mathematical score. They talk about string theory as being “elegant” or about the “beauty” of its mathematics. The most

striking thing about some of these “melodies” is that the ones that describe nature are, to theoretical physicists, like music of great beauty.

**D.** In the last lecture, there was an allusion to the fact that mathematics, at least as used by scientists, has pictures behind it, just as scores have music, rhythms, and melodies behind them. Today in our world, there are computers, which can be used as devices for “playing” mathematics.

1. The animations to be seen in these lectures are pictures of equations. The ideas come from equations.
2. It may be possible to think of computer-enabled conceptualization as a wave of the future, a new way for humanity to access mathematics. In this series of lectures, there will be heavy reliance on the fact that for the mathematics of string theory, instead of showing the mathematical scores to students, most of the time, the mathematics will be played on a computer.

**IV.** We begin with examples of this playing of mathematics, first with real physics and then moving into string theory.

**A.** The first example is relevant to understanding how the energy of the universe was distributed at the instant of the Big Bang.

1. In an animation, we see a grid with two lenses above it. Depending on how the lenses are shaped, the boxes on the grid look either smaller or larger than without the lenses.
2. Why is this relevant to the physics of the universe? The answer is that the amount of energy and matter that was distributed at the Big Bang acts like a lens.
3. In the first lecture, we considered the CMB, a remnant of the Big Bang, as an observation understood only through Einstein’s equations, particularly his *field equation*, which also describes the cosmological constant.
4. Among other things, this equation allows for an answer to the question: At the instant of the Big Bang, was there more energy in gravity, or did matter have more energy?
5. Knowledge of the CMB makes it possible to answer this question. Using Einstein’s equation, we find that the amount of energy in matter divided by the amount of energy in gravity acts like a lens.

6. If there were more energy in the gravity field, then the “visual static” in the CMB, seen as spots, would be larger than if there were no matter present. If there were less energy, the visual static, or spots, would be smaller. If the two quantities were equal, then the spot size of the lens should be exactly like the spot size that was predicted at the Big Bang.
  7. In our universe, the two sizes are exactly the same. It is possible to know then, that at the instant of the Big Bang, the amount of energy in gravity and the amount of energy in matter was the same.
- B.** Let’s look at a second example of how the computer can play a picture behind the mathematics that describes a fundamental part of our world.
1. In the last lecture, we noted that *evidence* of particles (the building blocks of atoms) has been seen for years in the data of laboratory experiments—and these same particles can be “seen” in mathematical equations—but humans have never been able to “see” them directly with their senses.
  2. An animation shows the nucleus of an atom, which is, in fact, a dynamic quantity. It’s not a hard sphere, as many imagine; it’s much more similar to a liquid drop. The little “balls” inside the nucleus are, again, things that, at first, had not been observed (and still have not been directly observed); there had been no way even to dream that such things existed.
  3. In 1969, an experiment was conducted at Stanford, called the *Deep Inelastic Scattering Experiment*; the only mathematical result that explained what was seen in the laboratory was one that posited small bits of matter, which we now call *quarks*, inside protons and neutrons. Again, mathematics enabled “seeing” something that there is no other way to “see.”
- C.** Our earlier discussion focused on pulsars and a wave of gravity that can travel from a pulsar. Before 1916, no one imagined this possibility. Now, Einstein’s equation of general relativity (GR) informs humanity that this is the case.
1. This has yet to be observed in the laboratory. The purpose of the Laser Interferometer Gravitational-Wave Observatory (the L-shaped buildings with arms about 4 kilometers long) is to detect these waves of gravity. How difficult is this detection?

To answer that question, a return to the picture of the helium nucleus is helpful.

2. Remember that the size of the nucleus itself is  $10^{-15}$  meters. The width of the red line in a picture is 1/1000 of the size of the nucleus of a helium atom. The waves of gravity coming from space will cause one arm of that long L-shaped building to stretch or shrink by 1/1000 the size of the nucleus of the helium atom. (This is the actual size of the shaking rather than the exaggeration used in the animation.)
  3. There has never been an instrument capable of measuring a change in distance of that size, so we must rely on Einstein's mathematics as a guide. The U.S. has now spent about \$400 million in the effort to allow humanity to see if this mathematics is supported as a valid view of nature.
- D. Many think that mathematics has nothing to do with their lives, but one final implication of GR theory (the theory of general relativity) should dispel this.
1. GPS units take into account that the radio signals traveling from the GPS unit to satellites that circle the Earth and back have their frequency shifted. Timing changes for the signals occur, and a correct description of these is provided by GR theory.
  2. It is *not* possible to design properly a GPS unit unless this mathematics is taken into account. Thus, this complicated mathematics, which often seems so esoteric and has nothing to do with non-scientists, is something easily encountered in everyday life. And usually such mathematics leads to something nice, such as a GPS unit, so that on a hike, you can avoid getting lost.

### Readings:

Burger and Starbird, *The Heart of Mathematics*.

Dunham, *The Mathematical Universe*.

Gates, "On the Universality of Creativity in the Liberal Arts and in the Sciences."

Wigner, "On the Unreasonable Effectiveness of Mathematics in the Natural Sciences."

**Questions to Consider:**

1. Give an example of the unreasonable effectiveness of mathematics.
2. What was Dirac's unexpected discovery?

## Lecture Three

### Apropos Einstein's Perfect Brainstorm Year

**Scope:** This lecture covers additional important concepts in physics that form a foundation to build insights into string theory. Continuing on a developmental path, the next two lectures will be devoted to particle physics and the “denizens” of the particle zoo, which have been believed—since the 1950s—to be the most basic building blocks of matter. Even though string theory goes one step further—to postulate that what were thought to be basic particles are actually oscillating strings—it is still possible to speak of such particles as gluons, quarks, and leptons in our discussions of string theory.

### Outline

- I. This “foundational” lecture begins with Albert Einstein and considers the sources of his theory of general relativity (GR), which lead to belief in the Big Bang and understanding of the cosmic microwave background (CMB).
  - A. Einstein had written several seminal papers by 1905, including one that informed our species that space and time were flexible, that is, space could appear to shrink and time could appear to be made to slow down as one approached the speed of light.
  - B. The sources that gave rise to GR theory actually existed well before Einstein’s time. In fact, it is possible to single out four mathematicians whose ideas were the major sources: Bolyai, Gauss, Lobachevsky, and Riemann.
  - C. Before Riemann, mathematicians had thought that only one kind of geometry existed—the kind students learn in high school. This *Euclidean geometry* implies, as examples, that parallel lines never meet, that the sum of the angles of a triangle must be 180 degrees, and that the area of a circle can be related to the square of its diameter by the number pi and a factor of one-fourth. The only problem with these *facts* is that it’s possible to define geometries in which they’re not true.
    1. An animation of Euclid’s parallel lines shows that the two lines never meet.

2. An animation of a triangle shows that, properly measured, the angles of the triangle add up to 180 degrees.
3. What happens when the animation changes, by drawing two parallel lines on a sphere, such as planet Earth? Starting with two points at the equator and extending them directly north, we see that they meet at the North Pole.
4. Next consider the triangle drawn, not on a sheet of paper but on the surface of Earth. If the angles are accurately measured, the angles of the triangle will sum to more than 180 degrees.
5. Finally, consider a circle drawn on the spherical Earth.
  - a. Imagine that you are on the equator and you begin walking directly north. You reach the North Pole, then continue walking until you arrive at the equator on the other side of Earth.
  - b. Meanwhile, you have a friend waiting at the same point on the equator where you started. You both pick up an end of this long line that you've inscribed on the Earth and begin walking in opposite directions.
  - c. When you and your friend have each walked halfway around Earth, you've created something similar to a circle, except it has a dome. The area of this shape is related to the length of the original line that you walked.
6. Consider a simpler scenario.
  - a. You begin on a flat plane and start walking. At a certain point, you stop and begin walking around the middle point of the line that you just inscribed, holding that middle point fixed.
  - b. You have now described a circle, and the area of the circle is equal to  $\frac{1}{4} \pi \times$  the square of the length of the original line that you walked.
7. In the first scenario, you traveled from the equator on one side of Earth to the equator on the other side; then, holding the North Pole fixed, you rotated the line around the surface of the Earth. The second scenario was similar, except it took place on a flat plane. Comparison of the areas of the two resulting figures reveals that, while the processes used for creating both were the same, the area for the first figure is twice the area of the second!

- D. These differences in geometry are puzzling; there was a time when the elements of geometry, as given to us by Euclid, were taken as sacred.
  - E. The mathematicians mentioned earlier figuratively asked the following question: Is it possible that the properties of lines, triangles, and circles depend on the object on which one draws them? Riemann ultimately showed that the differences in rules for geometry when done on a flat plane versus a curved sphere (or any curved shape) are mathematically distinct. In fact, he wrote the mathematics that describes when the differences will occur, using what he called the *metric* and the *Riemann curvature tensor*.
  - F. When Einstein investigated the structure of the universe and the Big Bang, he borrowed Riemann's mathematics, once again showing the curious interplay between ideas initially thought to reside purely in the subjective realm of mathematics but later used to describe objective reality.
- II. Perhaps the most exquisite use of Einstein's GR comes in the form of understanding gravity. A review of this concept is useful.
- A. We start with an animation showing the orbit of Earth around the Sun. The Earth typically takes 365 days to complete this orbit, and the light emanating from the Sun, traveling at roughly 670 million miles an hour, usually takes between 8 and 9 minutes to reach Earth.
  - B. Recall that these animations are actually visual representations of equations. It is possible to remove the Sun from the equation. When this is done, an expanding ring of darkness appears, and the Earth, *at the same instant* the Sun disappears, goes out of orbit.
  - C. Typically, when something happens in one place in the universe, there is a time delay before that event is "known" in another place. With the equations represented by the first animation, as soon as the Sun disappeared, the Earth went out of orbit, with *no* time delay.
    1. This situation is an accurate representation of the mathematics that describes gravity as given to us by Newton. His equations for the force of gravity do not allow for a time delay.
    2. Yet if string theory is to work, all forces must have a time delay because they are to occur in a unified description. Clearly, when something happens at one location in the

universe, the effect is not known instantaneously everywhere else.

- D. Remarkably, even without accounting for the time delay, Newton's mathematics was quite sufficient to permit travel from the Earth to the Moon. The mathematics states that the force of attraction between the Earth and the Moon depends on the product of their masses divided by the square of the distance of separation. Note, again, that time does not appear in this equation.
- E. This same equation applies to the force of gravity between the Earth and the Moon, and a similar equation can be applied to the force of repulsion between two electrons.
1. High school physics teaches that objects possessing like charges repel each other and do so according to an equation that is similar to Newton's law describing gravity: The repulsion is equal to the product of the charges divided by the square of the separation distance between the two.
  2. Note that this equation doesn't allow for a time delay either, because it's essentially the same as Newton's equation. Does this mean, then, that light instantaneously transports information from one location to another? (Remember, according to this equation, there is no time delay.)
  3. The answer is no. Light is a manifestation of the *electromagnetic* force. It must have a time delay. In the animation in which the Sun was made to disappear, that time delay was present, because the sphere of darkness took some time to get out to the radius of the Earth. This means a known time delay exists for light, but one does not appear for gravity. What's going on?
  4. We know how to mathematically describe the time delay for the transmission of light signals, based on work done by James Clerk Maxwell. In 1873, Maxwell wrote four equations that have a time delay built into them.
  5. It is even possible to understand how this time delay works by looking at the work of another physicist, Richard Feynman.
  6. There are two electrons in the *Feynman diagram* shown. One electron might move through space and time and, at a certain point, send a *message carrier* to a second electron with the message that both electrons ought to be repelled. If the

message carrier travels at finite velocity, then there will be a time delay.

7. There is such a message carrier—the *photon*. The photon, the particle of light, is the message carrier for the electromagnetic force. It's the reason why that region of darkness took some time to expand outward in the animation of the Sun's disappearance.
- F.** In looking at Newton's work on gravity and Maxwell's discovery of a time delay in the electromagnetic force, Einstein realized, figuratively speaking, that in order to include a time delay for gravity, it must also have a message carrier.
1. Using Riemann's mathematics, Einstein wrote the equation for GR theory, and his equation does for gravity precisely what Maxwell's did for the electromagnetic force—it allows for time delays. This difference between Newton's and Einstein's views of gravity can be seen by looking at a slightly different animation.
  2. With a return to the earlier animation of the Earth's orbit, we can once again turn off the Sun, at least in the equations. However, this time we see that the Earth continues in its orbit, because it does not yet "know" that the Sun has disappeared.
  3. It is only when the ring of darkness reaches the location of the Earth that the Earth goes out of orbit. Thus, the time delay is part of Einstein's equations of motion. There is a message carrier, the photon, which is responsible for the time delay for light, as a second message carrier, the graviton is responsible for the time delay for gravity. Both travel at exactly the same speed...671 million miles per hour.
- G.** A great thing about GR theory is that it informs humanity that our universe is consistent in a way we did not realize prior to its creation. Nothing is instantaneously communicated from one place to another with no time delay. Even Newton suspected this in his great *Principia*. The theory of general relativity (like its predecessor, special relativity) tells us that space-time is flexible, that there is a message carrier (the graviton, not implied by special relativity), and that nothing physical can travel faster than the speed of light.

**III.** Building a mathematical theory of everything is like being a parent on Christmas Eve and confronting a toy whose box states, “Some assembly required.” Like the parent on Christmas, when physicists see the myriad of shining pieces in physics, at first they make no sense. A parent might be prompted to think, “How can the toy possibly need all these parts?” This lecture has begun laying out on the floor all the parts needed for a theory of everything.

**Readings:**

Bernstein, *Einstein and the Frontier of Physics*.

Bartusiak, *Einstein’s Unfinished Symphony*.

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Gribbin and Gribbin, *Annus Mirabilis: 1905, Albert Einstein, and the Theory of Relativity*.

Harmon, *The Natural Philosophy of James Clerk Maxwell*.

**Questions to Consider:**

1. What did Riemann discover about the differences between geometry done on a plane versus a sphere?
2. What was the significance for Einstein of Maxwell’s inclusion of a time delay for the electromagnetic force?

## Lecture Four

### Honey, I Shrunk to the Quantum World—Part I

**Scope:** In Lectures Four and Five, we will work through the particle zoo before heading off to look at the dilemma—involving black holes—faced by Stephen Hawking, which led physicists to look beyond the world of particles. To understand how the atom works, one must go to smaller scales and explore the world of elementary particles, the “denizens” of the quantum world. The next two lectures look at the mathematical properties of the denizens of the quantum world to try to achieve a broader picture of how their mathematical description is related to that of space and time.

### Outline

- I. An animation of a typical atom—an amorphous “blob” surrounded by other blobs—begins the discussion.
  - A. The blobs in the illustration represent electrons, which are peripatetic. It is as if they “buzz” around the nucleus of an atom.
  - B. Atoms, in fact, are surprisingly empty. Consider a single blood cell in comparison to your entire body; that’s about the difference between the size of the atom and the size of the nucleus. The nucleus of an atom is to the entire atom as a single blood cell is to your whole body. Atoms are mostly just empty space, yet they are most times thought of as solid, because atoms are the building blocks for molecules.
  - C. The idea that the atom has parts began with an electrochemist named Stoney and electroplating experiments he performed in 1874. He believed the electroplating process could be understood more clearly if he posited that the atom could be broken into pieces and that one of these pieces was much, much smaller than the other.
  - D. This was the first time in history that someone had the idea that atoms could be broken apart. In about 480 B.C., Leucippus of Miletus and his student, Democritus of Abdera, proposed that, beginning with ordinary matter and successively breaking it into smaller pieces, one would eventually reach a piece that could no

longer be divided. This was the *atmos*, the most fundamental entity in the physical universe, and by definition, it could not be broken apart.

- E. Thus, Stoney's idea was revolutionary: Atoms have parts just like machines. He eventually gave a name to the smaller piece—the electron. Interestingly, the Nobel laureate physicist Leon Lederman has stated that about one-third of the economic activity in the United States is in some way related to manipulating electrons. All computers, cell phones, most televisions, radios, and other communication devices rely on the motion of electrons. Thus, most of the country's entertainment industry depends on these same motions. Transactions at an ATM depend on the same thing, and the list at times seems endless.
- F. The first person to give an accurate picture of the atom was the physicist James Chadwick, who proposed that the interior of the atom held an incredibly massive, positively charged object, now known to be the proton. Later, a second inhabitant of the nucleus, the neutron, was found.

II. This picture of the atom was only reached through struggle and experiment, as that's the way science works. It turns out that this is not the end of the story. Around 1969, the Deep Inelastic Scattering (DIS) experiments performed at the Stanford Linear Accelerator Center (SLAC) found that protons also held objects inside.

- A. From one point of view, these objects were expected; they had been predicted and named—*quarks*—by theoretical physicist Murray Gell-Mann. For a deeper understanding of quarks, I recommend The Teaching Company's course *Particle Physics for the Non-Physicist: A Tour of the Microcosmos*. For our purposes, we're going to make a brief visit to this strange world.
- B. This world is populated rather copiously. In addition to the electron, nature makes a copy of the electron that is 200 times heavier but in all other ways is an electron. This is called a *mu particle (muon)*. Roughly 10 years ago, we discovered that nature produces another copy of the electron that is 1700 times heavier. This is the *tau particle (taon)*.
- C. Since the 1930s, there has been evidence of particles that constitute a different kind of copy of the electron. These objects are electrically neutral, and hence, they are named *neutrinos*. The

electron itself has a neutral copy called the *electron neutrino*; the muon has a neutral copy called the *muon neutrino*; and the tau particle has a neutral copy called the *tau neutrino*. All neutrinos are incredibly light. In fact, up until recently, scientists were convinced that neutrinos had no mass at all.

1. Through a stunning set of experiments performed principally at a facility in Japan called Super-Kamiokande, it was eventually learned that, in fact, neutrinos do have mass.
2. This is important since neutrinos are by far the most numerous objects in the cosmos. Even though an individual neutrino has a very small mass, their expected incredibly large number means they must contain a great deal of the mass of the universe.

### III. Let's review the properties of these particles.

- A. The electron, the muon, and the tau have charge. All the neutrinos are neutral. Every single one of these objects behaves like a spinning ball that spins at a rate that can neither speed up nor slow down. Physicists have a special name for this rate of spin, *h-bar*. The electron, the muon, the tau particle, and the neutrino all spin at a rate of  $1/2 \text{ h-bar}$ .
- B. Also as mentioned, in the interior of protons and neutrons are smaller bits of matter, the quarks. In a neutron, there are two down quarks and one up quark. In a proton, there are two up quarks and a down quark. It is believed that all nuclear matter is put together in this way.
- C. Leptons consist of the set containing the electron, its two heavier copies, and three neutral particles, for a total of six objects. Quarks also come in sets of six. Up and down quarks make up the interiors of protons and neutrons. There are also "charmed" and "strange" quarks and, finally, "top" and "beautiful" (also called "bottom") quarks. These varieties are called *flavors*; each flavor comes in three *colors*. The total number of quarks, then, is 18.
- D. One of the most interesting properties of quarks can be traced back to the discovery of the physicist Paul Dirac. Recall that Dirac wanted to write an equation to describe electrons, but in doing so, he also described the electron's antiparticle, the *positron*. It is now known that for every quark, there is also an anti-quark, because quarks also obey the equation found by Dirac.

- E. Quarks (and anti-quarks) spin, as electrons do, clockwise or counterclockwise at the rate of  $\hbar$ .
1. The fact that these particles spin at this particular rate has an interesting consequence; it contributes to being able to sit in a chair, rather than passing through it. The main reason a chair supports a person is that the electrons in the person's body are repelled by the electrons in the chair. This force is called a *contact force*, and its underlying cause is electrical.
  2. Suppose, however, all electrical charges were turned off. What would happen then? The chair would still not allow a person to pass through it, because identical objects that spin at the rate of the electron have the property that no two of them can occupy the same place at the same time. There's still a residual repulsion, which is called the *exchange force*.

IV. Other forces can be further explored.

- A. One familiar force is gravity. It is responsible for keeping people in their chairs, as well as for galactic formation and planetary motion.
- B. Another force in nature is called the *weak interaction*. The weak interaction is responsible for the fact that naturally radioactive materials glow, principally through a process called *beta decay*.
- C. Also widely known is the electromagnetic force, which allows humanity to develop high technology and explains such phenomena as static electricity and lightning.
- D. Finally, there is another form of nuclear energy, the *strong nuclear force*. This is the form of nuclear energy from which stars derive their energy for producing light and heat.

V. In an earlier lecture, time delays connected to gravity and electromagnetic forces were discussed.

- A. As indicated, Maxwell was the first person to build the time delay into physics when he found that electromagnetic forces are communicated through space by a message carrier, the photon, the particle of light.
- B. Einstein later introduced the message carrier for gravity, called the *graviton*, or the *metric*.

- C. The weak nuclear force has three message carriers, collectively called *intermediate vector bosons*. Their individual names are *Z-zero*, *W-plus*, and *W-minus*. These bosons are very massive, and because they are so massive, the weak nuclear force can extend over only a very short range.
  - D. The strong nuclear force, the energy source for stars, has eight message carriers, called *gluons*. They are all massless, and they are responsible for holding quarks in the interior of the proton.
- VI.** To understand how message carriers work, a picture from a previous lecture may be used. Two spots of light represent electrons, and a wiggly line in the middle represents the message carrier, which in this case is the photon.
- A. The message carriers for the other three forces work in the same way as the photon in this picture.
  - B. For the gravitational force, the graviton is the message carrier that tells two objects they ought to be attracted to each other, because gravity always attracts.
  - C. For the strong nuclear force, the spots of light don't represent electrons; they represent quarks in the interior of nuclear matter, held together by the message carriers called gluons. For the weak nuclear force, the message carriers are the *Z-zero*, the *W-plus*, and the *W-minus*.
- VII.** Before ending this lecture, let's review some other astonishing aspects of the particles.
- A. In our world, time flows only in one direction. Everything moves toward the future (though Einstein's theory of special relativity implies not all at the same rate). But in the particle world, this is not true.
    - 1. The denizens—the quarks, the leptons, and the message-carrying particles—are capable of traveling forward or backward in time.
    - 2. Physicist Richard Feynman discovered this surprising property. He wrote equations describing the electron and found that they were perfectly reasonable if the electron was interpreted as moving backwards in time. However, when an electron moves backwards in time, it looks like its antiparticle, the positron.

- B.** Particle physics allows for the exchanging of charge. The laws of physics of the particle world allow an electron to be interchanged with its mirror particle, the anti-electron or positron. This swapping action is called *charge conjugation*, or *C-parity*.
- C.** Finally, imagine doing an experiment by looking at a mirror, making measurements, and writing out equations for what goes on in the mirror. It turns out that if the weak nuclear force is eliminated, the picture of the experiment seen in the mirror and the data coming from the experiment will be identical. This is called *parity reversal*. The physics of the microcosm, if you ignore weak nuclear energy, is *parity invariant*.
1. These properties collectively are called *CPT*: *C* for charge reversal, *P* for parity, and *T* for time reversal.
  2. One of the greatest mysteries in physics is the following: The physics of the microcosm is such that, in combining all three—doing an experiment in a mirror, watching a film of the experiment running backwards in time, and having all particles replaced by antiparticles—it turns out that the equations for that situation are identical to the equations that describe our world. This is called *CPT invariance*.
  3. *CPT invariance* is still one of the greatest mysteries of physics, because there are no ideas on why watching a backward-running movie in a mirror where particles have been replaced by antiparticles should lead to the exact same rules of physics in our microcosmic world!

### Readings:

Close, Marten, and Sutton, *The Particle Explosion*.

Oerter, *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*.

Suplee, *Physics in the 20th Century*.

Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics*.

### Questions to Consider:

1. What six particles associated with the electron are present in a lepton?
2. What are message carriers, and how do they work?

## Lecture Five

### Honey, I Shrank to the Quantum World—Part II

**Scope:** In the last lecture, we began a journey into the quantum world that is to continue in this lecture. In imagining being shrunk down to the level of  $10^{-10}$ , the realm of the atom, we encountered a number of different entities, including quarks, leptons, and force carriers. This lecture continues the exploration of the quantum world, highlighting aspects of this world that are very different from the classical world we live in.

#### Outline

- I. This lecture begins with a picture of a *meson*, a bubble-like object with a quark and anti-quark inside. Quarks are never free to roam the world as electrons do. Whenever a quark travels, it must be inside one of these bubbles, which are called *hadrons*. One kind of hadron is the meson, which contains quark–anti-quark pairs, but there is another type of hadron that contains quarks in triplets (an example of this kind of hadron can be seen in a picture of the nucleus of a helium atom). To date, no other type of hadron has been seen in experiments.
- II. The discovery of quarks followed a path similar to the discovery and categorization of elements in the history of chemistry.
  - A. In 1871, Dimitri Mendeleev began to impose order on the “zoo” of chemical elements in the form of the periodic table. Not all the elements in the table were known when he began to order them, and he used the missing pieces of the chart to predict that new elements would be found.
  - B. In 1964, Murray Gell-Mann and George Zweig looked at the profusion of nuclear matter and found a zoo; they imposed order on the zoo and made predictions based on the regularities they found in the properties of the particles. These predictions were eventually verified.
- III. In the last lecture, we observed that all the quarks and leptons spin at a certain rate,  $1/2$  times  $h$ -bar.
  - A. The carrier for the electromagnetic force has a spin rate of  $1$  times  $h$ -bar, i.e., twice the rate of the electron. The graviton, the

gravitational force carrier, has a spin rate that is four times that of the electron.

- B.** These facts about spin relate to the idea that identical fermions cannot occupy the same place at the same time.
1. If an attempt is made to push two identical fermions together (even with all electrical charges turned off), they are repelled. This is the *Pauli exclusion principle*, named for physicist Wolfgang Pauli.
  2. Imagine having control of light to the degree of being able to cause light particles, *photons*, to collide. Our animation shows that, unlike fermions, two identical photons pass right through each other. If the charges of the bosons could be turned off, they would show the same behavior.
  3. This represents an important lesson about the quantum world: In this world, not all particles behave in the same way with regard to the Pauli exclusion principle. Fermions naturally repel each other, and if all other charges are turned off, they still repel each other. Bosons repel each other if they have charge, but if the charge is turned off, they are allowed to pass right through each other.
  4. Scientists are starting to learn how to take advantage of this behavior in the attempt to build a new generation of computers. Present-day computers are based on electronics (i.e., manipulating electrons), but future computers may be built on photonics and use a process called *quantum entanglement* to trade information.
- C.** In the next animation, we see an atom, surrounded by electron clouds drawn as lobes. The lobes are held in position around the exterior of the atom by charge, and even if we turn the charge off, they remain exterior to the atom. If all the fermions in the animation are changed into bosons, however, all the electron lobes collapse onto the atom. This would be a monumental disaster for life, because chemistry relies on the existence of the electron lobes, and without chemistry, there is no life.
- IV.** A force carrier not mentioned thus far may be the most mysterious and important of all. This final boson is called the *Higgs particle*.
- A.** The table of particles lists various properties, including the masses of up and down quarks, charm and strange quarks, and beautiful

and top quarks, ranging from 1.5 MeV (a set of units used by physicists) up to 178,000 MeV for the top quarks. The mass of the electron in this same set of units is about .5 MeV, whereas the top quark is about 1.7 MeV.

**B.** Among the force carriers, some are massive, such as the W-plus, W-minus, and Z, and some have no mass, such as the photon and the gluons (the message carriers for quarks). Finally, there is the Higgs particle, which Leon Lederman has called the “God particle,” partly because of what it seems able to do in the universe.

1. In the animation, a particle with very little mass proceeds across the screen, passing through a series of circles that represent the effect of the Higgs boson. As the particle passes through the circles, it gains in size, representing an increase in mass. This is a graphical way of explaining how scientists envision the presence of the Higgs boson causing mass to arise in the quantum realm.
2. One of the primary purposes of the Superconducting Super Collider (SSC), a facility that *had* been proposed in the United States, was to detect the Higgs particle, but this facility was not built. Another facility, the Large Hadron Collider (LHC) in Geneva, Switzerland, will come on line soon and is expected to be capable of detecting the Higgs.

**C.** The bosons, then, are force carriers and are responsible for holding fundamental objects in their shapes. The fermions are the objects themselves, the stuff of which our universe is built.

**V.** This structure at the particle level leads to a number of interesting concepts, including the idea of *quantization*.

- A.** A basic definition of quantization is that energy exists in discrete, small packages, not in a continuum.
- B.** In the previous lecture, the force carriers were envisioned as small objects that carry messages from one electron to another or from one quark to another; objects should either be repelled or attracted. In the quantum world, this process is much more complicated.
- C.** In the quantum world, particles are capable of disappearing and appearing almost at will. In fact, it has been said that, in the quantum world, anything that can happen must happen, with some probability.

1. Sometimes the probability of an event's occurrence is very small and sometimes it is very large. The things that are most likely to happen in the classical world are also most likely to happen in the quantum world, but there are some differences.
2. One of these differences is in how the forces work. Our animation shows two electrons, one of which emits two photons. One of these photons is the message carrier that acts between the two electrons; the other photon is emitted, then absorbed by the first electron. This second photon has not sent a message to the second electron to be repelled, but it has carried some message to the first electron.

**D.** More complicated processes, such as *vacuum polarization*, exist. For this quantum process, the first electron sends a message carrier, which proceeds through space and time, but at a certain point, it disappears. However, real objects can't just disappear; in particular, the energy carried by this force carrier can't disappear. What happened?

1. A particle and antiparticle pair replaces the original force carrier. This pair is represented in the animation by the small circle in the middle of the diagram.
2. A particle and an antiparticle have opposite charges, which will attract each other. As seen in the animation, the two begin by traveling in opposite directions, but they are pulled together by their attraction. When they meet, they re-collide and disappear, because they are particle and antiparticle.
3. We must ask, of course, what happened to the energy the two were carrying? The energy is inherited by a second photon, which then serves as the message carrier to the second of the two electrons.
4. This whole process is a consequence of quantum mechanics, in which the way things occur is not always the simplest way they could happen.

**VI.** The quantum world is also different from the classical world in other ways.

- A.** In our world—the classical world—to throw a ball over a wall requires it to have enough energy from the throw to get the ball up and over. If a person could shrink down in size and try to throw an electron over a similar shrunken wall, the electron could get to the

other side of the wall with even less energy needed to throw it over the wall. This happens through a process called *quantum tunneling*.

- B.** In another example of the strangeness of the quantum world, a pitcher throws the ball in a smooth curve that is called the *classical path*, and the batter uses the classical path to try to hit the ball.
1. For the miniature pitcher who is able to throw an electron, under most circumstances, the electron will follow the classical path to home plate. It is also free to deviate from this path. The electron may veer off toward third base, orbit around third base 137 times, then travel to home plate.
  2. This scenario sounds like something from a cartoon, but real objects can exhibit these kinds of behavior (and do, since the proper operation of a cell phone depends on this). In the quantum world, the rules of nature are different from the rules in our world.
- C.** In another example, if someone is in a room with an open door and calls out, it is possible for someone who is outside the room and around the corner to hear them. Why is that?
1. Sound is a type of wave, and waves do not have to travel in a straight path, as baseballs do.
  2. When a sound wave reaches a barrier, it can bend around the barrier in a process called *diffraction*.
- D.** In the 1930s, two physicists, Werner Heisenberg and Erwin Schrödinger, used this analogy to understand the strange behavior of electrons. They wrote equations similar to those that had typically been used to describe waves, but they applied the mathematics to the behavior of electrons. With these equations, they were able to make predictions that describe why atoms have the structure they do.
1. If energy is applied to an atom, it will radiate at a very specific spectrum. The idea of applying the equations usually associated with waves to the behavior of electrons yielded this discovery.
  2. Again, using the idea that electrons have properties similar to waves, further discoveries were made about the structure of the radiation—what physicists call *fine structure*.

## VII. Why is the word *quantum* used to describe this strange world?

- A. Waves are usually thought of as being supported by some medium, such as the surface of water. Waves can be either gently undulating, or on a stormy day, the waves can be stirred up, and droplets of the water can actually be separated from the main body of water; one might think of a separated droplet as a “piece” of water. The word *quantum* means the same thing—a piece of something.
- B. Max Planck is credited with discovering that *quantization* is an aspect of our universe.
  - 1. The spectrum of light radiated by an object as it cools after being heated has a particular pattern. The original calculations made by scientists about this pattern, however, yielded opposite results to what was seen in experiments.
  - 2. In his work, Planck showed that light is not a continuous wave but comes in discrete packets, that is, *quanta* of energy. When this modification in thinking is made to the mathematics, one finds that the mathematical description of how an object radiates energy as it cools down agrees exactly with what is observed in the laboratory.
  - 3. In previous discussions, the notion of the quantum was used without discussion. There were discussions of the force carriers, for example, which are also associated with energy. The force carriers are, in fact, discrete packets of energy.
- C. At this point in time, the concepts of the quantum world constitute the most thoroughly tested area of science in existence. Physicists are absolutely convinced that our world, at its most fundamental level, is quantized.

### Readings:

Close, Marten, and Sutton, *The Particle Explosion*.

Oerter, *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*.

Supplee, *Physics in the 20th Century*.

Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics*.

**Questions to Consider:**

1. How does the Pauli exclusion principle apply to fermions and bosons?
2. Explain the concept of quantization.

## Lecture Six

### Dr. Hawking's Dilemma

**Scope:** This lecture discusses a crisis in theoretical physics that forced the physics community to think about quantum gravity more seriously than it had before. String theory is currently the only known resolution of this crisis. The problem won't be fully explored until later in the course, but we spend time here looking at the crisis in some detail.

### Outline

- I. In the last lecture, we discussed quantization, the idea that energy is never quite continuous and, at a fundamental level, must come in discrete packets. Energy is also associated with the gravitational field, and this energy must exhibit the same properties.
  - A. Though humans have never seen the effects of a quantum of gravity in the laboratory, this lecture will begin to take steps in that direction.
  - B. Gravitational energy is all around, sort of like a sea. In the real ocean, creatures such as whales are capable of communicating over tens of thousands of miles using frequencies that are attuned to wave propagation in the ocean. The successful operation of LIGO will enable humanity to hear the analog of “whale-song” generated by stars, galaxies, and cataclysmic events in the universe.
- II. Recall how gravity works.
  - A. Physicists write equations to describe how gravity works, such as an equation for tossing a ball into the air. The name of the parameter that describes how fast the ball leaves a hand is *initial velocity*. The greater the initial velocity, the higher the ball goes into the air.
  - B. Again using mathematics, one can find an initial velocity so great that when the ball is thrown into the air, it doesn't come back down. On the Earth, this speed is about 25,000 miles an hour and is called the *escape velocity*. When a ball is thrown with the escape

velocity, it has sufficient energy to escape the gravitational attraction of Earth.

- C. Imagine an astronaut on a small asteroid where the force of gravity is much less than it is on Earth. On this asteroid, she could throw a ball into orbit. Of course, if the mass of the asteroid were increased, the astronaut would have more difficulty throwing the ball into orbit.
- D. Escape velocity does not depend *only* on the mass of the planet. If one were to retain the mass of the planet Earth but shrink its radius by a factor of 4, then the escape velocity would increase to 50,000 miles an hour.
- E. According to Einstein's theory of special relativity, however, there is a maximum speed with which anything can travel in our universe. If the Earth was shrunk to the point where the escape velocity was greater than the speed of light, it would be impossible to launch anything into space with an escape velocity. This line of thought leads to one of the strangest objects considered by physicists, black holes.

**III.** The concept of a black hole was first suggested John Michell in 1783.

- A. A black hole is a body in which the linear density, that is, the size and the mass together, is so great that it requires an escape velocity greater than the speed of light. It is impossible to launch objects with an escape velocity from a black hole. For the Earth to become a black hole, its radius would have to be about 4 inches.
- B. As a star burns out its nuclear energy, there is nothing to stop its gravitational force from collapsing, if it is massive enough, into a single point called the *singularity*. Any star that is about four or five times more massive than our Sun will ultimately become a black hole.
- C. The significance of the singularity is that a black hole has the dimensions of a geometrical point (i.e., no size at all).
- D. In addition to the singularity, a black hole also has a sphere surrounding it, called the *event horizon*, which has the property that a light ray shot exactly tangent to the sphere would go in orbit around the black hole.
- E. Nothing inside the event horizon can escape; anything outside the event horizon is unaware of the existence of the black hole.

- IV.** In the late 1960s, Stephen Hawking began to wonder about black holes.
- A.** Nothing inside a black hole's event horizon can escape, but that's not true for energy, accounting for some interesting circumstances.
  - B.** An earlier animation showed a Feynman diagram of the exchange of force carriers between electrons. In this animation, imagine looking at the Earth and the Moon instead of electrons. The particle of light (the quantum of the electromagnetic force) between the two bodies, carrying the message they ought to be repelled, is replaced with the quantum of the gravitational force, carrying the message they ought to be attracted.
  - C.** Feynman was able to translate such diagrams into equations, using what are called *Feynman Rules*. Application of the Feynman Rules to the animation leads to the finding that the force of attraction between the Earth and the Moon is the product of the mass of the Earth and the mass of the Moon divided by the square of the distance between the two. This equation corresponds exactly with Newton's laws.
  - D.** In the quantum universe, this result is the most likely possibility, but it's not the only possibility. The Feynman Rules can also be applied to the picture that included a quantum loop to show that the force of attraction between the Earth and the Moon is infinite. Obviously, this result is not true; thus, on combining the two pillars of modern physics—quantum theory and relativity—the result is nonsense.
  - E.** Since the 1930s, theoretical physicists have tried to come up with a way to explain this problem. One explanation: Since space-time can bend and because none of the other forces is associated with the bending of space-time, perhaps the laws of quantum theory don't apply to gravity.
  - F.** The physicist Jacob Bekenstein argued that energy could be trapped within the event horizon of a black hole and that the laws of quantum mechanics likely hold true in this situation.
    - 1.** Bekenstein considered the idea of energy as trapped waves inside the event horizon. For a given amount of energy, the waves behave as if they have a certain order—called *entropy*—associated with them. Whenever a system possesses entropy, it also possesses heat.

2. Bekenstein discovered, then, that the waves trapped inside the event horizon have entropy; thus, they must also have heat. But the first law of thermodynamics says that if an object has heat, it must radiate, and we know that nothing can get out of a black hole.
  3. Stephen Hawking wrote a famous paper to explain how heat can escape from the event horizon. It turns out that under the laws of classical physics, in which there is no concept of the quantization of energy, there is no way for heat to escape. But using equations that grew out of the work of Heisenberg and Schrödinger—the same equations that govern electrons—heat can escape through the process of quantum tunneling.
- G. Hawking’s paper on black hole radiation does not conclusively prove his theory. It contains brilliant mathematics at the beginning and end, but in the middle, Hawking must insert his intuition in the form of equations called *grey-body factors*. His work is, thus, an exceptional example of Einstein’s famous dictum that imagination is more important than knowledge.

### Readings:

Greene, *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Miller, *Empire of the Stars: Obsession, Friendship, and Betrayal in the Quest for Black Holes*.

### Questions to Consider:

1. Summarize Stephen Hawking’s dilemma and his solution to it, that is, quantum tunneling.
2. Describe the properties of a black hole.

## Lecture Seven

### I'd Like to See a Cosmos Sing in Perfect Harmony

**Scope:** Before Hawking, astronomers believed that, looking at the night sky, when a black hole happened to pass through a line of vision, its only effect would be the distortion of star light. After Hawking, it was understood that black holes do more than just bend light; they also radiate heat. In other words, they operate consistently with the laws of thermodynamics. To reach this new understanding of black holes, scientists must combine two pillars of physics, quantum theory and GR.

### Outline

- I. An animation illustrates one concept of the theory of general relativity (GR).
  - A. In this animation of the Earth orbiting the Sun, we can see undulating lines, or indentations in the fabric of space-time; these are representations of the Earth's and the Sun's gravitational fields.
  - B. The mathematical function used to describe these indentations is called the *metric*. This is the same function that Riemann used to talk about the differences in geometry on a plane and on a sphere.
  - C. At this point in the exploration, there was only Stephen Hawking's mathematics (and imagination) to describe the physics of black holes, but there was no model to back up the mathematical imagination.
- II. Hawking's dilemma in the 1960s seemed totally removed from the problems associated with the structure of hadronic matter.
  - A. Experiments in particle and nuclear physics collide matter under extreme conditions and observe the products of those collisions to make deductions about the structure of matter. Some experiments conducted to understand the structure of hadronic matter had a curious result.
  - B. The outputs of some proton collision experiments in the 1960s were found to be strikingly similar to the inputs. Indeed, the plots,

of energy versus spin, of the outputs line up on straight lines, called *Regge plots*.

- C. Gabriel Veneziano successfully discovered the mathematics that would explain these straight-line plots. He found, by applying the *beta function* (mathematics from the 18<sup>th</sup> century), he could come up with an explanation. Yet he could not understand why the beta function should appear. Other physicists deduced that the appearance of the beta function could be explained if the outputs of the collision experiments were envisioned as strings.
- D. Note that this description of nuclear physics doesn't rely on quarks; in fact, quarks and strings were, at the time, competing models in physics. The Deep Inelastic Scattering Experiment seemed to settle the matter in favor of the quark model in nuclear physics.
- E. But the curious mathematics of string theory did not just go away. Even if it didn't describe nuclear physics, it wasn't necessarily wrong.

III. String theory diverges from several hundred years of physics, because its most fundamental objects are tiny filaments. The rest of classical physics is based on the notion that an understanding of everything in our universe can be achieved by observing a game of billiards.

- A. If you write the mathematics that describes the motion of billiard balls on a table, you arrive at Newton's second law. Indeed, all of our physics has been based on the idea that there are things in our universe that behave like little balls. In physics, the little balls are called *geometrical point particles*.
- B. Quantum theory replaced the point particles with waves, but the mathematics to describe these waves still depends on point particles.
- C. The theory of special relativity doesn't contradict the idea of point particles. Instead, it says that, in studying the motion of these little balls, you must take into account not just length, breadth, and thickness but a fourth dimension—duration, or time.
- D. String theory was the first conception of physics that was not based on little balls but filaments. As these filaments move, they produce incredibly complicated waves of vibrations. This is a property that no tiny ball can replicate.

- E. The physicist John Schwarz described the notion of the string as a “radical conservative reformulation of physics.”
- IV. In physics, when a new idea is proposed, it must agree in some cases with previously accepted theories. The earlier theories will generally apply up to a certain limit; then, the new theory works outside that range. In other words, the new, larger theory must be capable of connecting to the predictions of its predecessor.
- A. As known, Newton’s laws work on one level, but to describe the motion of electrons, you must appeal to quantum theory. Can you say that string theory agrees with previously accepted laws?
- B. This question can be put in a different perspective: In our universe, according to the laws of special relativity, nothing can travel faster than the speed of light, which in a vacuum, is 671 million miles per hour. Imagine a light beam that had been traveling for a period of 1 trillion years times the age of the universe, which is 14 billion years.
1. Now take this number and divide by your height; the result is roughly the size of a string, and it’s called the *Planck length*.
  2. The value of the Planck length is found from  $h$ -bar,  $G$  (Newton’s constant for gravity), and the speed of light<sup>3</sup>.
  3. Do these incredibly tiny strings obey the laws of special relativity?
- V. String theory is intrinsically quantum mechanical. When talking about little balls (points), the laws of Newton apply. With the laws of Schrödinger and Heisenberg, points are replaced by waves, but the location of the waves still depends on the points. With the laws of quantum mechanics, the world starts to look a little different, perhaps even weird.
- A. One of the weird aspects of quantum mechanics is *zero-point energy*, which can be explained as follows: Imagine a pendulum given a hard push; it will swing widely. If pushed more gently, it will have a narrower swing, and any degree of swing in between is possible. If not pushed at all, the pendulum should hang vertically with no motion. That’s a classical description of the behavior of a pendulum, or a *harmonic oscillator*, as it’s called in physics.
- B. A quantum pendulum, in contrast, can swing at one distance, or it can swing at other discrete distances, but not in between. There are

quantized regions of swinging. This behavior explains the spectrum of atoms: Electrons can orbit at one radius, or they can orbit at discrete larger radii, but not in between.

- C. Zero-point energy is an intrinsic part of quantum behavior, and because strings are quantum mechanical, they must have zero-point energy. They are never completely at rest.

**VI.** Objects in our world have definite patterns of vibration, and one can learn more about strings by looking at these.

- A. An animation shows a pipe with open ends that has a line running through it. The line represents the average velocity of air molecules. If the animation is set in motion, we can see a line vibrating up and down in a manner that is similar to a string. This motion is, in fact, an illustration about sound.
- B. If we look at a pipe with closed ends, we can see that the line vibrates, but the end points stay in the same location. In a closed pipe, the air cannot move outside the pipe.
- C. The mathematics of open strings and closed strings is the same as the mathematics of open pipes and closed pipes.
  - 1. A closed pipe actually has more motion associated with it, because at least two waves are needed, going in opposite directions, to keep the end points fixed.
  - 2. In an open pipe, the ends are free to move up and down; the number of vibrations needed to support this movement is not as great.
- D. This same behavior is seen in the mathematics of strings in relation to particles. The open string has fewer particles associated with it—but why are we talking about particles?
  - 1. Strings are too small to produce sound when they vibrate; from a human perspective, what they produce are particles.
  - 2. This is the secret of string theory: Things that to humans seem like different particles correspond in a string to different ways of vibrating.
  - 3. If a string vibrates in one manner, it seems to have the properties of an electron. If it vibrates in a different manner, it has the properties of a quark. All the denizens of the particle physics world, from the perspective of string theory, correspond to one object that vibrates in different ways.

- E. Open strings can describe photons or objects closely related to photons but not gravitons. Closed strings have more modes—in order to keep the ends fixed—thus, the objects associated with closed strings contain a mathematical description of the graviton.
- F. Two physicists, Joel Scherk and John Schwarz, noticed in the mathematics of string theory that there was an object with exactly the properties of the metric that Einstein had formulated. This is another name for the graviton.

**VII.** In this lecture, points were replaced with strings, and we came to an understanding that these strings vibrate just like real strings do in our world. But they don't produce music—they produce elementary particles.

- A. A challenge to face is whether these strings obey the laws of special relativity. This will be addressed in the next lecture.
- B. Thus far, string theory is the only consistent mathematics known to physicists that allows black holes to radiate heat in precisely the way that Stephen Hawking had to use his intuition to predict.

### **Readings:**

Ashton, *Harmonograph: A Visual Guide to the Mathematics of Music*.

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

### **Questions to Consider:**

1. What kind of experiments do physicists conduct to study nuclear matter?
2. Describe the progression from Newton's laws relating to "little balls" to special relativity, quantum theory, and string theory.

## Lecture Eight

### Einstein's Hypotenuse and Strings—Part I

**Scope:** How are physicists able to think about extra dimensions? The answer can be found in applying the mathematics of our world to a hypothetical world with other dimensions. In this lecture, the Pythagorean Theorem will be used for this purpose. Physicists don't necessarily visualize extra dimensions but, instead, calculate properties of objects in higher dimensions. In some cases, this is simply a matter of extending mathematics that is already well known.

#### Outline

- I. Our discussion will be using the Pythagorean Theorem to see how physicists calculate properties of objects in higher dimensions.
  - A. The Pythagorean Theorem states: The sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse.
  - B. In the animation, different colored lines are used as the sides of squares. Because the lengths of the green line (base of a square) and the blue line (top of a square) are initially the same, the areas of the squares are the same (an equation for this scenario implies that the area of the green square equals the area of the blue square).
  - C. Raising the green line, without changing its length, creates a new line. The square associated with this line is colored brown. As the green line was raised, the length of the blue line became smaller in order to remain directly under the end of the green line.
    1. The equation now shows that the area of the green square equals the area of the blue square plus the area of the brown square.
    2. No matter how the orientation of the green line is changed, this area equation holds true. This simple picture illustrates the Pythagorean Theorem.
  - D. We can relate calculating area to buy carpet for a room to the operation of squaring. To determine the amount of carpet needed,

multiply the length of the room by the breadth of the room. If the room is a square, this multiplication leads to the squaring operation.

- E. The initial animation was lying flat on a plane, but now the animation shows the initial line lifted up off the plane and brought forward in three-dimensional space.
  - 1. This leads to four squares; how did the fourth (purple) square arise? When the green line was lifted, it ended a certain height above the ground, which means it is possible to construct one more square.
  - 2. The equation now tells us that the area of the green equals the area of the blue plus the area of the brown plus the area of the purple.
- F. We return to a simpler scenario, before the green line was lifted off the plane. In this situation, there were only three squares. To describe this scenario in terms of mathematics: The area of the hypotenuse equals the area of leg 1 plus the area of leg 2, or  $H^2 = L_1^2 + L_2^2$ . In the animation, the area of the hypotenuse was the area of the green square; leg 1 was the blue square; and leg 2 was the brown square.

**II.** A new animation using a ladder against the side of a house reemphasizes these points.

- A. Initially, the ladder is used to construct two squares, one above the ladder and one below, much the same as in our first diagram. Since the ladder is the base of one square and the top of the other, the two areas are the same. When the ladder is leaned against the side of the house, the blue square shrinks in size and a brown square is created, but the sum of those two areas is still equal to the area of the green square.
- B. Placing the ladder perfectly vertically along the side of the house, the blue square disappears, and only the area of the brown square is equal to the area of the green.
- C. What happens if we realign the bottom of the ladder so it is no longer in line with the top edge? As before, another distance has been added: There is the distance from the base of the ladder to the side of the house; from the ground up to the top of the ladder where it touches the house; and the distance the ladder has been realigned at the top and bottom.

- D.** Earlier, algebra was used to write expressions representing these scenarios. With the ladder against the house, there was the equation:  $H^2 = L_1^2 + L_2^2$ . In realigning the top of the ladder, the equation changed:  $H^2 = L_1^2 + L_2^2 + L_3^2$ . Thus, in going from two dimensions to three, all that happened was to add one more area. In going to four, five, or six dimensions, the addition of the areas of more squares is indicated.
- E.** The way to study extra dimensions is by first trying to uncover patterns or rules in the mathematics of three dimensions, then changing the rules to accommodate more dimensions.

**III.** There is another way to think about these same ideas.

- A.** In our world, there are three independent directions in which we can move: front to back, left to right, and up and down.
- B.** If there were four directions in our world, there would be four areas in the Pythagorean Theorem. And if the four-dimensional world was our world, then the Pythagorean Theorem would be rather similar to one for the three-dimensional world.
- C.** In talking about extra dimensions in string theory, physicists are really talking about mathematical equations that have some attributes that are remarkably similar to the equations in our three-dimensional world, but that allow for the possibility that extra directions might exist.
- D.** Some of the mathematics of our world is such that with a change in the number of directions, the equations look the same. This property is called *invariance*. The idea of making changes to equations but leaving some things the same is called *symmetry*.

**IV.** It is also useful to consider volumes in a similar discussion.

- A.** Recall that the distance from the center of a sphere to its side is the radius. The formula to find the volume of a sphere is: Cube the radius, multiply that result by pi, multiply that result by 4, and divide by 3.
- B.** The volume of a cube is calculated by finding the product of its front-back measurement, left-right measurement, and up-down measurement. In a three-dimensional world, these three numbers are multiplied to find the volume of a cube. This suggests that to

calculate the volume of a cube in four dimensions, one needs to multiply four numbers.

- C. Mathematicians and physicists have known how to calculate the volumes of objects in mathematical universes of more dimensions since the time of Leibniz and Newton, the inventors of calculus.
- V. The full story of string theory lies *only* in its mathematics; this lecture shows how mathematics is the only firm guide to think about worlds with extra dimensions.
  - A. Do these extra dimensions really exist? The answer is not actually known, but string theory allows for them.
  - B. The challenge is to combine the two ideas from science—special relativity and quantum theory—with the theory of strings, or pasta. As we will see, this combination is the gateway to extra dimensions.

### **Readings:**

Abbott, *Flatland: A Romance of Many Dimensions*.

Mlodinow, *Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace*.

### **Questions to Consider:**

1. How would you calculate the Pythagorean Theorem in 16 dimensions? You can put your answer in words or mathematical symbols.
2. Is the idea of extra dimensions the stuff of science fiction, or is it somehow related to our world? In what way?

## Lecture Nine

### Einstein's Hypotenuse and Strings—Part II

**Scope:** In this lecture, we will consider how Einstein incorporated the fourth dimension of time into the Pythagorean Theorem and came up with an idea (here to be called), *Einstein's hypotenuse*. This leads to the famous equation  $E = mc^2$ , which turns out to be a statement about areas in a four-dimensional world. We close with a discussion of how Einstein's hypotenuse and string theory led to an object that could destroy the possibility for string theory to describe the world of physics.

#### Outline

- I. A graphical understanding of the Pythagorean Theorem shows a relationship among the areas of squares associated with a ladder that remains valid no matter how the ladder is oriented.
  - A. When the top of the ladder was out of alignment with the bottom, a final square was created that represented the distance the ladder was pushed to the side. Still, relationships among various areas remained valid.
  - B. Careful study of the Pythagorean Theorem for a world in which there are only the usual three dimensions implies how it should work in a world of more dimensions, The area of the square attached to the hypotenuse is equal to the sums of the areas of the squares attached to each leg in each distinct direction.
  - C. In other words, one area is added for each “extra” dimension that exists, and still the sums of these areas must add up to the area of the original square attached to the hypotenuse.
- II. In philosophy, literature, and other disciplines, the idea of time as a fourth dimension preceded Einstein. H. G. Wells's conception in *The Time Machine* possesses two subtle errors compared to Einstein's final insight.
  - A. According to Wells's conception, the idea of time as an extra dimension in the Pythagorean Theorem implies simply adding an extra area, as before. However, measuring lengths versus times requires different units, or *engineering dimensions*. One cannot

add square units of length, such as inches, to square units of time, such as seconds.

- B.** Einstein deduced a way around this problem by noting that light has both kinds of units attached to it. Light has a velocity of 982 million feet per second. The square of a velocity is meters<sup>2</sup> divided by seconds<sup>2</sup>. In special relativity, Einstein observed that the area of a square constructed from a unit of time multiplied by the square of the velocity of light results in a product with the units of square area.
- C.** When dealing with a duration of time, it must be multiplied by the speed of light to find a number that has units of length.
- D.** Einstein's new version of the Pythagorean Theorem, where the area related to time has the factor of  $c^2$  associated with it, reads  $H^2 = c^2$  times the area associated with a duration of time and then minus the areas associated with the three usual directions.
- E.** In the ordinary Pythagorean Theorem, the area of one square is equal to the *sum* of the areas of the other squares. Einstein, however, was one of the first physicists to assert that we could measure something very similar to a hypotenuse, but doing so required both adding and subtracting. For this reason, we will call this construct *Einstein's hypotenuse*. His choices of additions and subtractions are precisely required so that the speed of light is the same for all observers.

**III.** Looking at the ladder against the house, the area of the square associated with its length did not change, no matter its orientation. This property is called invariance. The secret to understanding special relativity is that Einstein's hypotenuse is invariant. How can we put this idea to use?

- A.** Imagine a light bulb first turned on for 9 seconds, then turned off. From the perspective of someone standing next to it, the light bulb doesn't move. If a second observer were in a spaceship traveling by at  $4/5$  the speed of light as these events occurred, both the person standing next to it and the light bulb would appear to be moving.
- B.** From the perspective of the first observer, the light bulb didn't move; therefore, all the areas associated with lengths are 0, but it is possible to construct an area using  $c^2$  that is associated with the

time the light was on. From the point of view of the observer on the spaceship, the light bulb is moving in space and it's on for a period of time. Before Einstein, scientists would have said the light bulb is on for the same amount of time for both observers.

- C. According to Einstein, however, the light bulb will appear to be on to the observer in the spaceship in such a way that the result of calculating Einstein's hypotenuse will give the same result for both. From this perspective, the light bulb moves a certain distance, and therefore, it is possible to construct an area associated with that movement. It stays on for some period of time; thus, square that time and multiply by  $c^2$ . These two numbers can be substituted into Einstein's hypotenuse.
- D. This calculation of Einstein's hypotenuse can be compared to that of the first observer, in which all the areas associated with lengths are 0. According to Einstein, the results must be the same. The only way for the results to be the same is that the time the light bulb is on according to the moving observer must be different from the time according to the observer standing next to the bulb. In fact, the moving observer will say the light bulb was on for 15 seconds; that's the number needed to make both results of calculating Einstein's hypotenuse agree.
- E. This scenario explains the invariance of special relativity. Measurements of space and time can be exchanged but must be traded in such a way that Einstein's hypotenuse is always the same.

IV. Einstein further realized that his notion of the hypotenuse not only applies to area and time but also to energy and momentum.

- A. For a scientist, momentum is the product of an object's mass and its velocity. If an object is at rest, it has no momentum.
- B. Energy is more difficult to define. In a sense, energy is the ability to do work, an idea made precise only with further concepts from physics.
- C. Momentum is similar to length; thus, if it is squared it yields something similar to an area. Einstein did this, then treated energy in a way similar to the treatment of time, with a difference: Energy squared divided by the speed of light squared yields a result in exactly the same units as the areas associated with momentum. The

areas of the momentums associated with three dimensions (left to right, front to back, and up and down) are then subtracted from this result for a fourth dimension.

- D. When an object is not moving, its momentum is 0, because it has 0 velocity. If we set the momentum to 0 and perform the calculation for Einstein's hypotenuse, we get something familiar: mass squared times the speed of light squared = energy squared divided by the speed of light squared. This is where  $E = mc^2$  arises in physics; it is Einstein's hypotenuse applied to an object at rest.
  - E. Einstein's ideas similar to the Pythagorean Theorem can be applied to our world, which has three dimensions of space and one dimension of time. To do that, one must multiply time by  $c$  in order to calculate the area associated with time. Einstein further implies that for the Pythagorean Theorem to work in our world, a generalization of the theorem using both addition and subtraction must be used. Using both is what permits the speed of light to appear the same to all observers.
  - F. Einstein's work informs us that these ideas extend to energy and momentum, where three of the lengths are associated with momentum and the fourth length is energy divided by the speed of light. The area of this hypotenuse is equal to mass times the speed of light. This result, combined with a momentum of 0, leads to  $E = mc^2$ . Thus, our world's most famous equation is a statement about areas in a four-dimensional world!
- V. Einstein's arguments never took into account quantum mechanics. To combine a quantum object, such as a string, with the idea of Einstein's hypotenuse requires a check to see whether the calculations still work.
- A. This was first done in modern physics in 1971–72. To apply Einstein's hypotenuse to the mathematics of string theory, accounting for quantization, 22 additional dimensions are required! Even in working with these 22 additional dimensions, one of the vibrations of the strings satisfies Einstein's hypotenuse but with the wrong sign for the square mass. This negative-mass-squared particle is called the *tachyon*.
  - B. If the tachyon existed, it would wreak havoc in the world of physics; an object whose square mass is negative could never come to rest—it would always have to move at least as fast as the speed of light.

- C. Further, the tachyon allows for probabilities that cannot exist in the quantum world.
1. Recall that the quantum world is one described by probabilities.
  2. In our world, if something definitely happens, it has a probability of 1. If something never happens, it has a probability of 0.
  3. In the quantum world, an event might occur, not with a probability of 1 but with some smaller number, such as  $1/2$ . This means that half the time the event happens, and half the time it doesn't.
  4. Such outcomes are allowed in the quantum world, but the probabilities must always fall between 1 and 0.
  5. However, with a tachyon in the quantum world, the probabilities can be greater than 1 or less than 0. In this situation, the mathematics for the quantum world doesn't describe anything; it is mathematics that has no physical interpretation.
- D. The tachyon is a kind of monster. If it existed and obeyed the rules of quantum theory, it would destroy the universe and tear down the foundations of modern physics. In 1971, when the first generation of string theory was explored, the tachyon was found, but if string theory was to be a true description of the universe, the tachyon would have to be locked away in Pandora's box.

### Readings:

Wells, *The Time Machine*.

### Questions to Consider:

1. How was Einstein able to add the fourth dimension of time into calculations of lengths in the Pythagorean Theorem? What is the name of the resulting construct?
2. What's the difference between probabilities in our world and the quantum world?

## Lecture Ten

### Tying Up the Tachyon Monster with Spinning Strings

**Scope:** In the first generation of string theory (called *bosonic strings*), as a quantum theory, the rules of probability must apply. The string is more than a particle; it vibrates, emitting what can be called “notes” or *modes*; from our perspective, these notes look like particles. To reconcile the probabilities for most of the notes of the string, we can add 22 extra dimensions in addition to those known to Einstein. The tachyon, a negative-mass-squared particle, threatened to destroy the mathematical viability of string theory as a description of nature. This lecture describes how physicists began to contain this monster and moved into a second generation of strings.

#### Outline

- I. Recall that the denizens of the quantum world all spin at a fixed rate: Electrons and quarks spin at a rate of  $1/2 \hbar$ .
  - A. The first generation of strings contained “notes” that spin at a rate that is 0, 2, 4, 6, 8... times the spin rate of the electron. In fact, these strings can spin at a rate that is equal to any even integer multiplied by the rate of spin of an electron.
  - B. In the quantum world, spin is an important quantity. For example, two particles that spin at odd integers times the basic rate, such as two electrons, cannot occupy the same space; they will repel each other even without the presence of charge. Objects that spin at any even integer times the rate of the electron can occupy the same space at the same time. This is the difference between fermions, in the former case, and bosons, in the latter case. Thus, in the first generation of strings, every note—every oscillation of a string—is a boson, hence its alternative name, the *bosonic string*.
  - C. The mathematics of our world describes more than bosons. The electron is a fermion, not a boson. The first generation of strings could not describe our world, since there was nothing in its mathematical structure that possessed the attributes of leptons or quarks.

- II.** To solve this problem, we need to look at the mathematics of fermions.
- A.** A spinning bicycle wheel may be suspended between a person's arms. If the wheel is spinning in one direction, the right hand is closed; if it is spinning in the other direction, the left hand is closed.
- 1.** An arrow can be painted on the ground below the closed hand pointing away from the wheel to indicate the direction of spin. A second person away from the wheel looks at the arrow, then looks away, and completes a full circle around the wheel. Upon returning to his original place, he looks again to find the arrow pointing in the same direction, assuming nothing touched the arrow in the intervening time. For mathematicians, this arrow is called a *vector*. You must walk completely around the arrow before it is seen to point in the same direction as when initially viewed. There are things in our universe that don't have this property. A ball is one. If placed on the ground, a ball, when viewed from any angle, appears the same. For mathematicians, these kinds of objects are called *scalars*.
  - 2.** Many of the denizens of the quantum world have similar properties. The photon behaves like a vector. The Higgs particle behaves like a scalar. Electrons and quarks have different properties from either of these.
- B.** Consider the spinning bicycle wheel again. If the arrow was observed, then walked around once and found pointing in the opposite direction, you would think a trick was being played. What if you walked around twice and the arrow is found pointing in the original direction?
- C.** There are objects in our universe, called *spinors*, that behave in just this way. If you walked once (or any odd number of times) around a *spinor*, it would be found to point in the opposite direction from an initial viewing. If you walked around it twice (or any even number of times) and then viewed it, the spinor would point in the original direction. Nothing disturbs the spinor during the period of the walks!
- D.** In 1913, a mathematician named Élie Cartan was the first person to write a mathematical description of this strange behavior, using a construct known as *Clifford algebra*.

- E.** In the 1930s, Wolfgang Pauli, later joined by Dirac, showed that this strange behavior described in mathematics can be used to describe objects in nature.
- F.** In 1921, two scientists, Otto Stern and Walther Gerlach, performed an experiment to measure the spin of electrons by directing an electron beam through a path lined with magnets.
1. Since Maxwell, scientists knew that a charged spinning object acts just like a bar magnet whose properties depend on the spin rate. Two bar magnets repel each other if placed north pole to north pole, but will attract each other if placed south pole to north pole.
  2. Since spinning objects behave as magnets, an electron beam is equivalent to a beam of bar magnets. If a path is lined with magnets, the electrons will be pushed around by magnetic attraction or repulsion.
  3. By measuring the beam deflection in this experiment, they measured the rate of spin of the electron.
  4. Stern and Gerlach found the electron spins at a rate of  $\hbar$  in a positive sense or in a negative sense, but never at any other rate.
- G.** These laboratory results showed that rates of spin are quantized. Further, these results enabled the confirmation of the flipping behavior of spinors. All electrons and quarks exhibit the behavior of spinors!
1. This behavior is the secret of chemistry and explains why two electrons can exist in the lowest energy shell of an atom: One electron has its spin up, and the other has its spin down.
  2. At the next level of the energy shell of an atom, eight electrons can exist, four with up spin and four with down spin.
- H.** It is counterintuitive to consider that objects that have no size behave like they spin. Due to the Stern-Gerlach experiment, we know this property is valid for all fermions (leptons and quarks).
- I.** Spin is ubiquitous in the quantum world. All electrons and all quarks spin at a rate of  $1/2 \hbar$ . Photons, the intermediate vector bosons, and the gluons all spin at twice the rate of the electron. The Higgs particle does not spin at all. These are all force-carrying bosons.

- III.** For physicists investigating particles during the 1970s, the concept of spin was well known, but it presented problems for the first generation of string theory as the mathematics did not account for spins for fermions.
- A.** Recall in first generation strings, the vibrations (or notes) were seen to occur with spin rates that are only any *even* integer times the rate of spin of an electron.
  - B.** Three physicists, Andre Neveu, Pierre Ramond, and John Schwarz, brought a new perspective to the problem of spin and strings around 1971. Their innovation was to include a mathematical solution for string theory that describes *odd* integers times the rate of spin of the electron, including the integer 1. The resulting mathematics allows for the electron.
  - C.** With this innovation, physics moved from the first generation of the string (the bosonic string) to a second generation (spinning strings). The spinning string has electron-like objects. Some spinning string notes obey the Pauli exclusion principle and others do not. The force carriers do not obey the Pauli exclusion principle and the objects the forces act on do. Thus, the second generation of string theory looks more like our world.
- IV.** The tachyon was still present in this second generation of string theory, destroying the world of mathematical physics.
- A.** Though the string has an infinite number of vibratory modes, only the tachyon with its negative-square mass wreaks this destruction.
  - B.** The tachyon is the result of a calculation performed to reconcile probabilities in the quantum world. The mathematics for its exorcism from spinning string theories goes back to a remarkable Indian mathematician named Ramanujan, who performed deep investigations into number theory. His equations described the absence of probabilities outside of 0 and 1, about 70 years before string theory was invented.
  - C.** Given that the presence of notes associated with even-spin-rate objects was not sufficient to neutralize the tachyon, maybe the notes from the odd-spin-rate objects would do so. It turns out that the “extra” spinning notes have a property associated with them that allows the removal of the tachyon from the spinning strings.

- D. String theory in this second generation is almost safe from the tachyon. To deal with the tachyon requires another symmetry, as we had when looking at the Pythagorean Theorem and the ladder leaning against the house.
1. Among the notes of a string, bosons (like the force carriers) and fermions (like quarks and electrons) occur. Imagine the blue box from our first animation of the Pythagorean Theorem represents the bosons of a spinning string and the brown box represents the fermions. If for every fermion there was a boson and vice versa, perhaps figuratively speaking, it is possible to trade the size of the blue box for the size of the brown box and yet always maintain the analog of the size of the green box.
  2. This concept is called *supersymmetry*. It requires that mathematical solutions like that describing the electron can be traded for other solutions that describe objects like the photon. This idea is only 30 years old, and it involves even more esoteric mathematics. Further discussion of this idea will occur in future lectures—it turns out that the brown box has an associated minus sign, which will bind the tachyon and lead beyond spinning strings to superstrings.

### Readings:

Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*.

Kaplan and Kaplan, *The Art of the Infinite: The Pleasures of Mathematics*.

Maor, *To Infinity and Beyond: A Cultural History of the Infinite*.

Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite*.

### Questions to Consider:

1. What was the significance of the Stern-Gerlach experiment?
2. How does the second level of string theory differ from the first?

## Lecture Eleven

### The Invasion of the Anti-Commuting Numbers

**Scope:** This lecture takes us deep into the world of string theory. We have already met the tachyon, which threatened to destroy physics, and considered a way to banish it, ensuring that string theory is more like our world. An important ingredient in this progression was the inclusion of the attribute of spin into the mathematics of string theory. An even stranger property of spinors (possessed by all quarks and all leptons) is anticommutativity, which will be discussed in this lecture.

#### Outline

- I. For scientists who study fundamental questions about physical reality, it often seems as if nature is as teeming with mathematics as our planet is teeming with life.
  - A. Students are introduced to different number systems at various stages, for example, the counting numbers, the number 0, fractions, and the so-called *transcendental numbers*, the most familiar being pi.
    1. Counting numbers were likely among the first mathematical knowledge acquired by humans. The question “How much?” is not an abstraction when it is part of the issue of survival.
    2. This marked the first acquisition of *numeracy*, the analog of literacy—the ability to abstract numbers in the world around us.
    3. Ancient societies developed numbers, but many of these early systems did not include the number 0. The number 0 did not appear until sometime between 200–650 A.D.
    4. The idea of positional notation—our system of reading numbers from right to left—first appeared in Babylonia and India.
    5. It took almost 500 years for the Arabic-Hindu number system to become accepted in Europe. Fractions were not generally used in Europe until the 16<sup>th</sup> century, but there is evidence that Egyptians were using fractions in a base-10 system as far back as 1800 B.C.

- B.** At times, nature is not so instructive about numbers. This lecture is about another element of the mathematics of string theory that is critically important in the removal of the tachyon. It requires a little-known type of number, *Grassmann numbers*. To ease the discussion, we will first review another uncommon type of number, complex numbers, via their historical development.
- II.** In an animation, a pyramid with a square base is manipulated to form a volume called a *frustum*.
- A.** The frustum leads to a problem famous in human mathematical history.
1. Consider how to measure attributes of the frustum. The symbol  $B$  is used to represent the length of one side of the base, the symbol  $T$  to represent the length of one side of the top, and the symbol  $H$  to represent the height.
  2. The determination of the volume of the frustum is compared to the determination of the volume of a cube.
  3. Due to Newton and Leibniz, the volume of the frustum can be found using integral calculus. Remarkably, the formula for this volume also seems to have been known by the Egyptians.
- B.** The measurement of the height vertically down from the top of the frustum through to its base is difficult to obtain in comparison to a measurement of one of its slanted edges. The symbol  $S$  is used to represent the length of a slant edge, and we can relate the volume to this measurement.
1. The formula for the relation of the height measurement to the slant-edge measurement leads to an interesting problem. If the slant-edge term is smaller than the difference term, calculation of the volume involves taking the square root of a negative number. It took 1500 years to do this successfully.
  2. The square root of a negative number is not obvious. For example:  $-1 \times -1 = +1$ .
  3. Finding the square root of  $-1$  leads to *imaginary numbers*. A historical reference on imaginary numbers is described in a book called *An Imaginary Tale: The Story of  $\sqrt{-1}$*  by Paul J. Nahin.

### III. Let's review an ordinary multiplication property of numbers.

- A. Multiplication has the property of *commutativity*; that is, the order in which the multiplication of two ordinary numbers is performed doesn't matter. For example:  $3 \times 4 = 4 \times 3$  and  $3 \times 4 - 4 \times 3 = 0$ . We can also write this same idea using algebra:  $a \times b = b \times a$  or  $a \times b - b \times a = 0$ .
- B. Between 1844–1862, mathematician Hermann Grassmann proposed the concept of a new type of number to satisfy the equation that reads:  $a \times b + b \times a = 0$ . If  $a$  and  $b$  are the same number, then this equation says that a number times itself is zero. Among ordinary numbers the only solution to this is zero itself.
- C. *Grassmann numbers* have the property that the order of multiplication does matter, in other words,  $a \times b =$  (a positive number), but  $b \times a =$  (a negative number)!
- D. Ordinary numbers obey the equation  $a \times b = +b \times a$ , called *commutative multiplication*. Grassmann's numbers obey the condition  $a \times b = -b \times a$ , called *non-commutative multiplication*.
1. Measurements involving photons act like commutative multiplication. A measurement of the properties of two photons in the order photon<sup>1</sup> and photon<sup>2</sup> gives the same answer for the properties measured in the order photon<sup>2</sup> and photon<sup>1</sup>. As an equation, this yields: photon<sup>1</sup> × photon<sup>2</sup> = +photon<sup>2</sup> × photon<sup>1</sup>.
  2. If the same measurement process is applied to electrons (or any fermion), the resulting equation has a minus sign: electron<sup>1</sup> × electron<sup>2</sup> = -electron<sup>2</sup> × electron<sup>1</sup>. This minus sign is the source of chemistry, via the Pauli exclusion principle.
  3. The reason two fermions can't occupy the same place at the same time is, if they could, there would be no way to distinguish them. It would be as if both fermions were the same Grassmann number. The same Grassmann number multiplied by itself must give zero. Thus, a measure of two such fermions must give zero!

#### IV. What are the implications of Grassmann's rule to string theory?

- A. Recall that the spinning string has modes or vibrations. These can be separated into two classes: (a) those with the properties of bosons and (b) those with the properties of fermions.
- B. A tachyon can interact with both classes (bosons and fermions) of modes of the spinning string.
- C. In the quantum world, anything that can happen must happen. In particular, the effects of the tachyon on the bosons and fermions depends on including the effects of "loop" diagrams.
- D. Due to their anticommutativity, fermion loops generate minus signs compared to bosonic loops. This provides the key to elimination of the tachyon.
  - 1. Given a diagram with some structure of the loops, the mathematical expression that is calculated depends on which class of string modes appears in a loop. If a given answer is found using only bosonic loops, replacing one of the loops by a corresponding fermion loop gives the same answer *but* with the opposite sign!
  - 2. Thus, if the tachyon interacts with equal numbers of bosonic and fermionic loops, the net effect on the tachyon adds to zero. Correspondingly, the tachyon then has no effect on the paired bosons and fermions.
  - 3. In other words, if we set the number of boson vibrations equal to the number of fermion vibrations, the tachyon is contained!
  - 4. The simplest way to set these two numbers equal is through the  $N = 1$  *spinning string*. All of the modes of vibration in this spinning string satisfy Einstein's hypotenuse; that is, the bosonic/fermionic pairs of modes have attributes of the particles seen in our world. This pairing is the fundamental meaning of supersymmetry.
- E. In a previous discussion of how the modes of the bosonic string satisfy Einstein's hypotenuse, we noted the need for 25 spatial directions and 1 temporal direction. Physicists use the numbers 25 and 26 interchange-ably in this context. (In an earlier lecture, the number 22 was used to describe all the "extra" ones over and above the 4 used by Einstein.)

1. In this new generation of strings, having banished the tachyon using equal numbers of boson and fermion vibrations, the question of how to satisfy Einstein's hypotenuse must be revisited.
  2. It turns out that with equal numbers of bosons and fermions, only 9 spatial directions are required to satisfy Einstein's hypotenuse, the familiar 3 spatial directions plus an extra 6 directions. This is only true if *Kemmer angles* are not also introduced. (Kemmer angles will be discussed in Lecture Sixteen.)
  3. Therefore, the superstring, with its equal numbers of bosons and fermions, looks more like our universe. This was an enormous boost for string theory: It eliminated the embarrassing problem of having probabilities greater than 1 or less than 0. In superstring theory, all the probabilities are between 0 and 1, which is consistent with our view of the quantum world.
- F. There are more complicated ways to banish the tachyon; one of these is through  $N = 2$  strings. To satisfy Einstein's hypotenuse with  $N = 2$  strings requires two time directions and two space directions! This is nothing like our universe. There are even more ways to enforce the cancellations that banish the tachyon, but these resemble our world less and less and, ultimately, result in the return of the tachyon in a different guise!

**Readings:**

Nahin, *An Imaginary Tale: The Story of  $\sqrt{-1}$* .

**Questions to Consider:**

1. Explain anti-commuting multiplication.
2. How does anticommutativity help string theory?

## Lecture Twelve

### It's a Bird—A Plane—No, It's Superstring!

**Scope:** Fermions, such as the electron, have many remarkable mathematical properties. A strange property with respect to one or more electrons, anticommutativity, means that if the measurement of some property of a pair of electrons is made using one order of the electrons and the order is switched, the measurement of the same property gives the same result but with a minus sign. This property of electrons is the source of the Pauli exclusion principle and all of chemistry. In 1977, Gliozzi, Sherk, and Olive observed that it is supersymmetry, the equality of bosons and fermions, which banishes the tachyon monster from string theory. Green and Schwarz provided a better way to describe superstring theory which reveals even more superstrings.

### Outline

- I. In the everyday realm, *symmetry* means balance, but the definition in mathematics or physics is different. To illustrate symmetry, we return to a now familiar problem. The arena is the invariances associated with covering a square stage with carpet.
  - A. In a fanciful story, a gnome named Tachyon is introduced to discuss the concept of the action of a discrete symmetry. The properties of an area of 25 square inches under the replacement of 5 by  $-5$  (thanks to the magical powers of the gnome) illustrate the mathematical definition of a symmetry. In this fairy tale, the invariance prevents an observer from knowing when Tachyon has exercised his power.
  - B. Next, we consider continuous symmetry via use of a triangle. Our animation for the Pythagorean Theorem is used, with Tachyon making a challenge for the observer to guess the areas associated with each leg given only the information about the hypotenuse.
    1. Given only the area associated with the hypotenuse, it is impossible to guess the areas associated with the legs since the known orientation may be changed at Tachyon's will.

2. In physics, this is called *continuous symmetry*, meaning variables can be changed from one to another in small or large steps.
3. The idea of continuous symmetry is critical in nature; it is possible to write equations that accomplish these changes using quantities that mathematicians call *generators* (regarded as *exchangers*).

## II. Now let's turn to the idea of supersymmetry.

- A. As seen from lectures, there are two distinct classes of objects in our universe: (1) the building blocks of our world, i.e., fermions, and (2) the forces and force carriers that hold those building blocks together, i.e., bosons. Tachyon regards these like the areas associated with the two legs seen in the Pythagorean Theorem.
- B. Tachyon is then endowed to change only one property for all known ordinary matter: The rate of spin of a particle may be increased or decreased by  $1/2$ .
- C. If Tachyon changes the rate of spin of an electron, it is no longer an electron. Similar decreases in the rates of spin of all the fermions and bosons could be implemented to replace them with rates that differ by  $1/2$  a unit of spin.
  1. An example starts with up and down quarks inside of protons with spin rates of  $1/2$ . Tachyon could change the rate of spin of these particles to 0. The quarks would no longer be quarks.
  2. In our world, the carrier for the electromagnetic force, the photon, spins at a rate of 1. Tachyon could change this rate of spin to  $1/2$ . If the spin is changed, the photon is no longer a photon.
  3. If Tachyon's powers were real, he could double the amount of "stuff" that makes up reality.
- D. Recall the banishment of the tachyon required an equal number of bosons and fermions to interact with the tachyon. The imagined universe where the gnome Tachyon changes the spin of ordinary particles and force carriers would be exactly the kind of mathematical structure needed to eliminate the mathematical tachyon from string theory. This balance is what is meant by space-time *supersymmetry*.
- E. Physicists began to consider models that exhibited supersymmetry only in the late 1960s and early 1970s.

**III.** In the original bosonic string, supersymmetry was unknown, and introducing the idea of spin was not sufficient to banish the tachyon from string theory. That requires equal numbers of objects spinning at different rates. Thus, supersymmetry was born from the need to make string theory consistent.

**A.** Supersymmetry is such a strange idea that it actually doesn't require strings.

1. Our graphic shows the bosons and fermions of our world on the left-hand side of the screen. These bosons and fermions exist in the *standard model of physics*, a model supported by tens of thousands of experiments and which offers a precise description of nature.
2. Attempting to balance the standard model by positing the universe represented by the right-hand side of our graphic led to the *minimal supersymmetry standard model*, but this doesn't actually require string theory.
3. Supersymmetry, though required by string theory, could exist without string theory (if one also forgets quantum gravity).
4. This leaves the physics community with a conundrum. In a few years, the Large Hadron Collider (LHC) is expected to enable the search for evidence of supersymmetry in nature. If superpartners are discovered, string theory will get a boost, but this alone will not definitively prove superstrings describe our universe.

**IV.** We mentioned five superstrings; how did this number arise?

- A.** The open and closed pipes we looked at in a previous lecture, in some sense, capture the mathematics of string theory. Immediately, we think that there should be two superstrings, one captured by open pipes and one by closed. That's two of the five.
- B.** The Green-Schwarz superstrings are all defined by spinors appropriate for a world of 9 spatial dimensions and 1 temporal dimension. To avoid the re-introduction of the tachyon, we can ask how many of these spinors can be added to the bosonic string. The answer turns out to be two left-hand spinors, two right-hand spinors, or one of each. The mathematics of right and left spinors is the same, so we add two spinors, bringing our total to four superstrings. The mathematics consistent with adding two of these sorts of spinors requires the superstrings to be closed.

- C. A fifth superstring is also possible. It is a most remarkable object. It was “invented” in 1984 at Princeton by David Gross, Emil Martinec, Jeffrey Harvey, and Ryan Rohm. Called the *heterotic string*, it can, in some ways, be viewed as an extension of things already discussed.
1. The bosonic string had 25 spatial directions. With the addition of spin, the new superstrings had only 9 spatial directions.
  2. Suppose there was some way to meld these ideas together and get some kind of consistent mathematics. How many dimensions would be involved? This is a challenge for the heterotic string.
  3. When this challenge is successfully confronted, Einstein’s search for a unified field theory comes to an end. The fifth string is the direct answer to Einstein’s question.

**Readings:**

Greene, *The Elegant Universe*.

Kaku, *Hyperspace*.

**Questions to Consider:**

1. Can you think of a mathematical example of the property of symmetry that we haven’t discussed?
2. Describe supersymmetry using the graphic we saw in this lecture that showed fermions and bosons on the left and their superpartners on the right.

## Lecture Thirteen

### Gauge Theory—A Brief Return to the Real World

**Scope:** String theory is not yet science; it is mathematics. But the peculiar thing about the mathematics of string theory is that it looks like the mathematics we use to describe the real world. In some ways, the mathematics of string theory is inspired by known science, and it ought to begin to make predictions to be tested, but it is not at that point yet. In this lecture, the starting point is the year 1861, the year in which a young Scottish physicist, James Clerk Maxwell, first understood how we are able to see color. Maxwell was the physicist upon whose shoulders Einstein most clearly stood in producing the theory of special relativity. The ultimate unsuccessful quest of Maxwell and Einstein was the same: to find a unified field theory that would account for both the electromagnetic force and gravity.

#### Outline

- I. A paper Maxwell wrote in 1873, entitled “A Treatise on Electricity and Magnetism,” contains four equations which led to an understanding of light.
  - A. Maxwell’s equations imply that light is the propagation of an electromagnetic wave. These equations require photons to change as they move through both space and time in order to describe electric and magnetic fields of force.
  - B. To describe light, these fields can be depicted as “playing a game of leapfrog.” The electrical field changes in space and time and generates a magnetic field, which in turn, changes in space and time and generates an electrical field, and so on. The speed at which this occurs is the speed of light.
  - C. This was Maxwell’s great discovery, and all of our current communication technology can be traced back to these four equations.
  - D. In 1888, Heinrich Hertz constructed an experimental device to detect Maxwell’s waves of electromagnetic energy.

- E. Maxwell tried to describe gravity in mathematical terms 50 years before Einstein tried to modify his equations describing gravity to include electricity and magnetism. Both failed.
- II. The concept of *change* of the photon is important to understand electricity and magnetism (also collectively called *electromagnetism*).
- A. Our familiar animation of the Pythagorean Theorem is set in motion to show changes in the areas of the blue square and the brown square.
- B. A second animation with two bowls of ping-pong balls, one fuller than the other, shows that even if two objects are different, the change in each can be the same.
- C. Maxwell's equation implies a similar relationship between electricity and magnetism, which can be measured in a laboratory, and photons, which are not directly measurable. It is the change in photons which produces the electric and magnetic fields.
- D. The same electromagnetic fields can be produced by different photons, each undergoing the same change. Since the fields are produced by changes in photons, not by the photons themselves, many photons are indistinguishable. This is an important idea in physics, called *gauge invariance*. Different photons that produce the same electromagnetic fields are related to each other by a quantity (*gauge parameter* or *Kemmer angle*) that is similar to an angle in the real world.
- E. By imagining a box filled with photons, we can consider the question of how much energy in the box is due to the photons.
1. More than one photon can produce a specific electric or magnetic field. Thus, there could be two boxes in which the electric and magnetic fields were the same and which could possess the same electromagnetic energy, even though the photons in each box were different. The important point is that the changes in the photons could be the same.
  2. The energy in each box must depend on the photons in some way since, if there were no photons at all in the boxes, there would be no electromagnetic energy.
  3. The fact that it is impossible to tell which photon produces how much energy leads to the idea of *charge conservation*, first understood by Emmy Noether. The *Noether theorem* implies that if there are changes in a quantity that is not

directly measurable, but that some physically measurable quantities don't depend on those changes, then there must be conserved currents.

4. What is the conserved quantity associated with photons? The answer is electric charge. Due to Noether's theorem, it is understood that the conservation of electric charge is actually an implication about the ability of two (or more) distinct photons to produce the same electric or magnetic field.

**III.** In the quantum world, the laws of motion are very different from the laws of motion in our world. This has an important implication for charge.

- A. We consider a second animation which uses a bowl to compare motion in our world with that of denizens in the quantum world. In our world, balls set in the same position in the same way all follow the exact same path (a result of classical physics as described by Isaac Newton).
- B. If the balls are charged objects, this same picture gives an illustration of the conduction of current. All the current in—that is, the balls on the rim of the bowl—is equal to all the current out—that is, the balls falling into the hole. This illustrates charge conservation in our world.
- C. In a second animation, we use the laws of motion in the quantum world. If successive balls are set in exactly the same manner on the rim of the bowl, they don't all travel along the same path as a consequence of the laws of quantum theory. Still, one possible outcome is that all the balls still fall into the hole, which means that, even though the laws of motion are different, charge is still conserved.
- D. Quantum theory allows for a second outcome. Given that the balls can move along different paths, some might never fall out of the bottom. If some of the balls never get to the bottom, charge is not conserved.
- E. This *anomaly* was discovered in the 1950s: In a quantum mechanical universe, sometimes charge may not be conserved.

**IV.** These properties must be incorporated into the view from string theory.

- A. If charge is not conserved, then all of the photon has relevance, and this results in a *time-like particle* in the mathematical

description of the photon. Unfortunately, time-like particles act like tachyons (although slightly different than tachyons) and have the power to render all predictions of physics nonsensical. Thus, a theory with anomalies makes no predictions about the real world.

- B.** Any kind of force carrier on the quantum level may have an anomaly, not just the force carrier of the electromagnetic force.
  - C.** The presence of anomalies was worked out by three physicists, Stephen Adler, John Bell, and Roman Jackiw, and the resulting mathematics is called the Adler-Bell-Jackiw theorem.
  - D.** This theorem can apply to mass and energy. As shown by Einstein's great discovery, these may be regarded as different attributes of the same thing. It can also apply to other forces in nature, not just electromagnetism. Both the strong and the weak interaction can be subject to anomalies. And if an anomaly occurs, time-like particles appear for all the force carriers, totally destroying the quantum world and the ability to make calculations.
  - E.** So far, this has been a story about our world from the view of the denizens, but the mathematics of anomalies also occurs in string theory. The vibrations of a string produce, from our point of view, particles. If that's the case, then all the force carriers that we know in our world, which are actually vibrations of a string, can acquire anomalies, and the string itself must be subject to anomalies.
  - F.** Just as the presence of anomalies destroys our ability to make predictions in the real world, so, too, does this apply to the mathematical world of string theory. Time-like particles have exactly the same effect as tachyons: They result in mathematical descriptions in which probabilities can be greater than 1 or less than 0.
- V.** Around 1982, John Schwarz and Michael Green worked out the mathematics of supersymmetry to banish the tachyon. They also found that they could incorporate, into this concept, forces similar to the electromagnetic and other forces. Because the mathematical world of Schwarz and Green was quantum mechanical, however, it might also be subject to anomalies.
- A.** While working on the question of the relationship of anomalies to forces and force carriers in superstring theory, Schwarz and Green found something remarkable.

1. Since all forces have force carriers, they all have charges associated with them. The electrical force is associated with electrical charge; the strong nuclear force has color charge; the weak nuclear force has weak charge; and gravity is associated with the Newton constant. If there are many forces at work, then there must be many charges.
  2. In looking at the mathematics of something similar to our bowl analogy, Schwarz and Green found that only two mathematical constructs allow all the balls on the rim of the bowl to fall out of the bottom. One of these constructs required exactly 496 charges! From where did this number arise?
  3. Recall again the discussion of areas and invariance and our illustration of the Pythagorean Theorem. In two dimensions, the area of the green square is always equal to the area of the blue plus the area of the brown. A similar construct for these areas was given in three dimensions. Each of these areas can be thought of as representing charge, at least in terms of the mathematical calculations of Schwarz and Green.
  4. In our world, there are three basic rotations: about the z axis, the x axis, and the y axis. In string theory, one way to arrive at the number 496 is to consider a world in which there are 32 directions to rotate, and then just add dimensions, as in the conception of the Pythagorean Theorem for the original bosonic string. The number 496 is the number of rotations that can be constructed if the world had 32 directions. For this reason, the string that Schwarz and Green found in 1982 was called the *SO(32) superstring*.
- B.** The *SO(32)* string is a way to understand where the number 496 comes from: Almost magically, for 496 charges, all the charges make it to the bottom of the bowl, but for any other number, anomalies appear, which result in time-like particles, which in turn, destroy probabilities. Only using 496 in this special way, considering rotations, does a piece of mathematics emerge that might allow the ability to make predictions about our world. This was the birth of superstring theory.
- C.** In 1982, it became clear that the goal of Einstein was within reach.
1. Both Maxwell and Einstein tried to accomplish the same goals using different approaches, and both failed. Maxwell wrote

equations to describe electricity and magnetism, then tried to modify his equations to describe gravity. Einstein wrote equations to describe gravity, then tried to modify those equations to include electricity and magnetism.

2. The magical number 496 arose in the context of a string theory, and the difference within a string theory is that all theories describing closed strings necessarily describe gravity. On the other hand, strings are capable of vibrating in many different ways, and one of their other ways of vibrating describes the force carriers associated with forces like electromagnetism.
3. A theory that embodies in its mathematical structure the description of photons and the description of gravity implies success where both Maxwell and Einstein failed, and indeed, the world's first unified field theory was born in 1982 with the work of Schwarz and Green and the construction of the SO(32) string.

### **Readings:**

Harmon, *The Natural Philosophy of James Clerk Maxwell*.

Oerter, *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*.

### **Questions to Consider:**

1. What is charge conservation?
2. What was the problem with charge conservation in the quantum world before 1982, and how did Schwarz and Green solve the problem?

## Lecture Fourteen

### Princeton String Quartet Concerti—Part I

**Scope:** Humans are generally familiar with the properties of light. It has brightness (*intensity*) and color (*frequency*, or *wavelength*). Maxwell's great discovery that light is a kind of wave implied one additional property. In this lecture, this property of light, *polarization*, is discussed, and we will see how an analog of this property was used to construct a new kind of string.

### Outline

- I. A computer graphic shows light as a wave combining two different oscillating “objects”—more formally, *fields* (electrical and magnetic fields).
  - A. In the animation, the blue balls on the horizontal axis constitute an electrical field; the green balls on the vertical axis constitute a magnetic field. As a wave of light propagates, the two fields oscillate together. Thus, light is a dual vibration of electricity and magnetism.
  - B. Light has another property called *polarization*. To simplify this explanation, our discussion will ignore the magnetic field and concentrate only on the electrical field.
    1. We see an animation of a light ray with the characteristic shape of a wave. A crest appears on the left side of the image and a second crest on the right side. The distance between these two crests defines the *wavelength*, known as its color.
    2. The tip of an electrical field of a light ray can be described as it enters the eye of an observer. The electrical field lines might simply vibrate up and down. This is called *linear polarization*. The field lines might also rotate, which is called *circular polarization*. This can occur in either a counter-clockwise or clockwise sense.
    3. In the center of an animation we find a large circle with a line oscillating vertically to represent linear polarization. In linearly polarized light, the electrical field vibrates only along this single line. In the two smaller circles, the green arrows

rotate around the smaller circles to illustrate circular polarization.

4. The property of polarization was first discovered in the mathematical description of light, but most people cannot detect polarization with their eyes. In 1846, a geologist named von Haidinger noticed he could correlate a particular image with the polarization of light. This effect is called *Haidinger's brush*.
5. When people who are sensitive to polarization look at a bright sky, they see a diffuse pattern that looks roughly like two figure 8s—one vertically oriented and one horizontally oriented—caused by the polarization of light. Turning one's head to the side may cause the figure 8 to rotate, as shown in the animation. If the light is linearly polarized, the figure 8 doesn't move.

C. There are many devices to detect polarization. The polarization properties of light are used in polarized glasses to reduce glare.

1. An animation depicts an electrical field as balls lined up along an axis. If the electrical field is set in motion, a wave starts to form, but an object is blocking the wave. In this orientation, the light cannot pass the blocking object. If the orientation of the object were changed to exactly line up with the electrical field, the light could pass. This is the basic mechanism behind polarized glasses.
2. The reason polarized glasses cut down on glare is that glare typically comes off a surface with horizontal polarization; in other words, the electrical field is moving horizontally. The lens, then, can be oriented to block the horizontal polarization.

II. The rest of this lecture will show how an analog of Haidinger's brush was used, in 1984, to construct a new string.

A. Green and Schwarz looked at the mathematical properties associated with anomalies in the superstring and found the anomalies could be eliminated with exactly 496 charges.

B. This 1984 construction, which allowed for anomaly freedom in string theory, also presented a puzzle.

1. As stated in the last lecture, 496 is the number of rotations that one can perform in a mathematical world in which there are 32 different directions. Green and Schwarz also realized that,

in addition to the rotations around the 32 axes, there was another way to arrive at the number 496.

2. This other way was discovered by Élie Cartan, a mathematician who studied rotations and certain mathematical generalizations. In trying to find a description of the most general mathematical rotation, Cartan also found the most complicated rotation that one can describe using mathematical formulations. This rotation is called  $E_8$ ; the  $E$  stands for *exceptional*.
3. Rotations are described by angles; therefore, if we have 496 rotations, we must have 496 angles.
4. The mathematical object  $E_8$  is associated with exactly 248 rotations and, therefore, 248 associated angles. It is easy to see that  $248 + 248 = 496$ , the magical number that prevented the anomalies from causing havoc in superstring theory. One set of rotations had been found, but where was the other set?

**III.** Let's return for a moment to the  $SO(32)$  string, which played an important role in understanding string theory.

- A. Where exactly on the strings do these 496 charges occur? For the  $SO(32)$  string, the charges occur at the ends of the open type of string. Before 1984–1985, this was the only way that physicists knew how to describe a string that had charge; namely, the string carried charge at the ends of the filament.
- B. Why must the charges be restricted to the ends of the filaments?
  1. Some lectures ago, we noted the fact that a closed string has properties similar to a sound wave in a pipe. Recall that animation showed a sound wave as a bowed shape tied down at the ends of a closed pipe. In an open pipe, the ends of the wave were free to move. In string theory, this same mathematics occurs.
  2. The closed pipe corresponds to a string with the ends tied together. These kinds of strings always describe gravity. The open strings, the ones that have the charges on the ends, always describe photons, but they never describe gravity.
  3. A theory that describes both gravity and charges seems to require a string that combines both open and closed types, knowing that charges occur only on the open strings.

- C. In 1984, four physicists at Princeton University, Gross, Rohm, Martinec, and Harvey, who became known as the Princeton String Quartet, solved this puzzle. They became mathematically conscious that there was a kind of brush associated with open and closed strings. To understand their work requires a second look at open and closed strings.
1. An animation shows a closed string in motion. It oscillates, but there are points, called *nodes*, at which the string is not moving.
  2. The Princeton String Quartet realized that every mathematical description of closed strings written up to 1984 had the property that the nodes of closed strings remained fixed at the same locations.
  3. These fixed nodes are the analogs of linear polarization for real light rays in our world (the middle circle in the animation of polarization), where the electrical field vibrated vertically but never rotated.
  4. Understanding these nodes and their connection to linearly polarized light prompts a question: Is it possible to introduce into the mathematics of string theory objects that correspond to circularly polarized light? If so, it turns out that charges on the string need not be restricted to its ends but could be distributed throughout the length of the string.
  5. Consider a second animation of a string, with the nodes indicated by red dots. In this animation, the nodes move. The motion of the string roughly corresponds to left circularly polarized light; thus, the string is called a *left-mover*. Of course, we can also write the mathematics to describe *right-movers*.
- D. The Princeton String Quartet went a step further by writing the mathematics that took advantage of the second set of Cartan's rotations and found that the number 496 comes from two  $E_8$ s. This *heterotic string* can be interpreted as the logical combination of strings from different dimensions. The left-moving parts of the string describe a superstring (meaning it has spin associated with it), but the right-moving parts of the string describe the old bosonic string that had the tachyon problem. Remarkably enough, this synthesis works.

1. The supersymmetric part of the string, the left part, eliminates the tachyon, but the right part of the string is like an open string, which allows the charge to be distributed throughout the heterotic string.
2. Many physicists questioned how this synthesis could be accomplished mathematically, but in fact, the mathematics already existed in the physics literature, provided by two physicists, Makoto Sakamoto and Warren Siegel. The first one gave the mathematical description of *heterotic fermions*, and the second gave a different mathematical description—that seemed unrelated—of *chiral bosons*.

- IV. The description the Princeton String Quartet wrote to measure its energy does not contain any quantities that add to 496. Where were the charges?
- A. This part of the story is linked to a Scotsman, John Scott Russell, who in 1834, observed a strange solitary wave in a canal. This kind of wave, called a *soliton*, has some properties of particles; namely, two of these waves can bounce off each other.
  - B. The Princeton String Quartet argued that the charges that make up the magical number 496 are distributed between the ordinary waves of their theories and the mathematics that describes solitons.

### Readings:

Darrigol, “The Spirited Horse, the Engineer, and the Mathematician: Water Waves in Nineteenth-Century Hydrodynamics,” in *Archive for History of Exact Sciences*, 58.

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

### Questions to Consider:

1. Describe linear and circular polarization of light.
2. Explain the analog of polarization in string theory.

## Lecture Fifteen

### Princeton String Quartet Concerti—Part II

**Scope:** The initial work of the Princeton String Quartet resulted in a combination of two strings from different dimensions, a left-moving superstring and the old bosonic right-moving string. But this work did not directly take into account the 496 charges required for anomaly freedom. In this lecture, we'll attempt to find one more description of the heterotic string, based on angles associated with rotations, that helps us arrive at that magic number.

#### Outline

- I. The old bosonic string had 25 spatial dimensions and 1 temporal dimension. If we looked at Einstein's hypotenuse for such a string, the mathematical formula seemed very long and complicated.
  - A. Introducing supersymmetry, the form of Einstein's hypotenuse becomes much simpler. The number of lengths is reduced from 25 to 9.
  - B. Each one of the lengths in Einstein's hypotenuse is a mathematical attribute found in string theory. Describing the string as having 25 dimensions requires 25 mathematical "objects" to describe it.
  - C. In this way, the string is not a single strand. The old bosonic string had 25 plus 1 strands. The newer superstring has 9 plus 1 strands.
- II. Last lecture, we learned about the construction of the heterotic string.
  - A. The heterotic string combines left-moving, right-moving, and standing-wave modes. In the earlier animation of a standing-wave string, as it oscillates, note that the nodes, indicated by red dots, do not move. (Remember, *node* means a point at rest; *mode* is a way of vibrating.)
  - B. In addition to the standing-wave modes, strings have left- and right-moving modes in which the entire string oscillates and rotates. Both the modes and the nodes rotate as a single unit.
  - C. This behavior is an echo of real physics, known since the 1870s, when Maxwell included the property of polarization in his equations.

- D. Of course, the discussion of the 496 charges was included to avoid the charge-conservation anomaly, yet the Princeton String Quartet did not include 496 strands in their papers on the heterotic string.
- E. The 9 plus 1 strands apply only to configuration space. To review: In our world, an object can move front to back and requires one strand to describe this movement in its mathematics. An object can move left to right and up to down and requires one such strand to describe each of these motions. Movement through time, from past to future, requires its own strand. At a minimum, to describe our world, four mathematical “strands” are required, collectively called “the string.”
- F. To arrive at the magical 496 strands, the Princeton String Quartet had to appeal to a mathematical object called a *soliton*. These strange waves actually appear in nature; if two collide, they bounce off each other.
- G. If some strands describe space, is it possible to find a strand-like description of the charges in heterotic string theory?
  1. In the original papers of the Princeton String Quartet, there was no sign of such a description. In one of their mathematical descriptions, a set of 16 bosons and 480 solitonic waves appears.
  2. The original papers also introduced a description that used 32 fermionic strands and 464 solitonic waves.
  3. How can these two descriptions relate to the same mathematical object? Remember, bosons carry the forces in our universe, and the objects the bosons act upon are fermions. Bosons and fermions are not the same kinds of objects.

**III.** The answer to this question is related to some aspects of our world.

- A. In 1911, mercury was cooled for the first time to  $452^\circ$  below 0, where it loses all resistance to electrical current and becomes a *superconductor*. As electric current is sent through ordinary wires, part of its energy goes into heating the wire. In a superconducting wire, this does not happen, which has the potential to save a great deal of energy.
- B. Superconductors were not fully understood until Bardeen, Cooper, and Schrieffer wrote equations to explain how they work.
  1. In our world, electrical currents are transported by electrons.

2. The mathematical work on superconductors showed that, sometimes, electrons, even being fermions and obeying the Pauli exclusion principle, can join together as if they were a single object. The resulting object, in many ways, has the properties of a boson, an important fact that allows superconductivity to work.

C. If it were possible to combine an electron and a photon, mathematically, this object would behave like a fermion. If it were possible to join two photons together, the resulting composite would behave like a boson. The mathematics of these combinations is like a multiplication table.

1. Bosons and fermions in this multiplication table act like the numbers  $+1$  and  $-1$ . Thus,  $+1 \times +1 = +1$ , or boson  $\times$  boson = boson. Similarly,  $-1 \times +1 = -1$ , or fermion  $\times$  boson = fermion, and  $-1 \times -1 = +1$ , or fermion  $\times$  fermion = boson.

2. The behavior of the joint composites of these objects is determined by assigning  $+1$  for bosons and  $-1$  for fermions. These rules apply to any number of fermions or bosons joined to form a composite, e.g., a composite of three fermions acts as a fermion.

D. In our world, it is ultimately possible to determine whether these combined objects are made of two bosons or two fermions, but in the two-dimensional world of the string, this is not possible, because fermions can perfectly disguise themselves as bosons. This explains why the original work of the Princeton String Quartet had two different descriptions for the same theory: In one of the descriptions, the fermions join together to behave as if they are bosons.

IV. The original papers on the heterotic string include strands, but the number of strands associated with electric-like currents never actually equals to 496; it is the strands (not associated with motion in space) plus the solitons that equal to 496. This isn't completely satisfactory.

A. A simple way to find the influence of the heterotic string on all its vibratory modes that have the properties of the denizens is to "count" the amount of energy carried by each strand. Because the original descriptions of the heterotic string don't have all the objects necessary to make that count, there must be another formulation.

- B.** To get at this other formulation, we again use the example of the ladder and the house. The square sizes are changed by using different orientations of the ladder, starting with it as the base of one square and the top of another square. A third square appears by angling the ladder. No matter how we orient the ladder, the area of the green square is equal to the area of the blue plus the area of the brown.
1. We can write the Pythagorean Theorem to show that the area of the green remained the same, but this equation allows trades of area between the area of the blue and the area of the brown.
  2. In the first equation, when the ladder is on the ground, the first area is built using the length of the ladder; because none of the ladder is lifted up on the wall, the second area is 0. When the ladder is in the vertical position, the first area is 0, and the second area, using the length of the ladder which was 0, is now the length of the ladder.
  3. In the real world, a person is required to move the ladder. But there is a mathematical object whose role it is to take the ladder from the ground and place it against the wall of the house. These objects are called *generators for symmetry*. Recall that angles measure how much the orientation of the ladder is changed. The *exchanger* (or generator of the symmetry) is a mathematical representation of the person who implemented the exchange!
  4. For the heterotic string, we have 496 generators. (Remember that the heterotic string is an object that acts as if it has 32 different directions in which it can rotate.)
- C.** Sophus Lie created the general mathematical tools for describing rotations of related motions. The challenge is to use these tools to provide one more description of the heterotic string, one in which there are exactly 496 strands (in addition to the 9 spatial strands and 1 temporal strand necessary for position in space for the heterotic string).
- D.** The standard model contains force carriers, and each of the force carriers is associated with one of these generators. In order to make the heterotic string appear mathematically closer to our world, it must possess strands of the string that mimic gauge

parameters (Kemmer angles) associated with gauge invariance of the force carriers.

- V. Is it possible to find this last formulation? The answer is remarkably simple.
- A. The angles of the 496 angles associated with the  $E_8$  rotations can be interpreted as being the missing strands. However, in order to write consistent mathematics, we must identify the angular variables with the variables that describe the 496 strands.
  - B. The way to solve the missing-strand problem is to introduce 496 angles associated with right-movers of the string. No additional left-movers are required since these are already contained in the standing waves.
    1. In the animation of polarization of light, if the right-moving circle is added to the left-moving circle, one obtains exactly the circle in the middle—the depiction of linear polarization.
    2. The standing-wave modes of the heterotic string actually contain both left and right modes hidden within them, in the same way that the graphic of polarization contains both left-circular and right-circular modes in the linear mode.
    3. To consider the full version of the heterotic string, a series of additions is required. First, 10 of the middle pictures from the polarization graphic are needed, because those describe the 10-dimensional structure of the heterotic string. Next, 10 left-movers are required to provide the mathematical description of the spin of the heterotic string. Finally, 496 right-movers are needed to describe all of its charges.
    4. This is the mathematical structure of the heterotic string—the goal toward which this discussion has been leading; it includes the angles that look just like the angles associated with change of force carriers in the standard model.
- VI. In our last lecture, we talked about the fact that photons are not directly measurable; it is the *change* in photons that generates electric and magnetic fields, and that is what can be measured.
- A. We also said that two different photons could produce the same change, or the same electric and magnetic fields. What's the relationship between these two photons? They are related to each other by an angular-like variable; therefore, charge conservation must also be related to the appearance of such angle-like variables.

- B.** In our world, in discussions about the forces for electromagnetism, extra angles do not appear; we see the force carriers that are related to each other by fields we can measure. The ambiguity in the relationship between the “measurables” and the mathematical descriptions is what results in these angle-like quantities.
1. For the eight gluons in our world, there are eight angle-like quantities associated with gluons. Similarly, in our world, possessing one photon, there is one angle-like variable associated with the photon. Finally, there are three force carriers for the weak nuclear force; therefore, there are three angle-like variables associated with these force carriers.
  2. These angle-like variables actually occur in our world, but they don't refer to directions that we see. They refer to the fact that we are talking about a change in a photon or a photon-like object.
- C.** The now-solved puzzle of finding the last formulation of the heterotic string was one in which these angle-like variables could appear. This last formulation is my own research, conducted with Warren Siegel.
- D.** Let's summarize the three formulations of the heterotic string:
1. The original formulation of the Princeton String Quartet used fermions, the  $-1$  variables. The second formulation used bosons, the  $+1$  variables. But neither of these formulations included the 496 angles that make the heterotic string look like our world.
  2. Only in the third formulation can we clearly see that the exact same mathematical structures that describe the standard model are present in the mathematical structure of string theory. This gives us more confidence that string theory is connected to our world.

### Readings:

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Hoddeson and Daitch, *True Genius: The Life and Science of John Bardeen*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics*.

**Questions to Consider:**

1. What was missing in the first two formulations of the heterotic string that prompted the search for the third?
2. Summarize the three formulations of heterotic string theory.

## Lecture Sixteen

### Extra Dimensions—Ether-like or Quark-like?

**Scope:** After Maxwell, the scientific establishment believed that light, as a wave, propagated through a medium called the *luminous aether*, or *ether*. When scientists tried to measure the velocity of the Earth relative to the ether, they got null results. With Einstein, it was instead understood that time and space measured by different observers in different frames of reference yield different results, but it is only the relationship between the numbers that changes, not the object being measured itself. In string theory, the mathematics of the heterotic string can be interpreted in two ways. The most popular interpretation argues for extra dimensions. The alternative asserts that what may be thought of as angles arising from extra dimensions are actually angular (or angle-like) variables associated with the change of force-carrying particles. This interpretation lends itself to a much more flexible interpretation of string theory.

### Outline

- I. Before Einstein, the scientific establishment believed light, like all other known waves, propagated in a medium called the *luminous aether*, or *ether*.
  - A. With special relativity, we realized that the way in which space and time is measured depends on how this is done *relative* to some motion. An important consequence of Einstein's idea was that even though measurements of an object may appear different to different observers, the object itself is not actually changing. The properties of an object that remain unchanged are described using Einstein's hypotenuse.
    1. A line segment, stretched in one direction, could be measured with a yardstick to be 3 feet long. If the yardstick was oriented perpendicular to the line, the measurement would be 0. The two numbers (3,0) describe the point at the end of the segment. If the line segment were re-oriented  $90^\circ$  with respect to its original direction, the two measurements could change to 0,3.

2. Notice that nothing happened to the line segment; only the mathematical descriptions of the position of its tip changed.
  - B. Einstein realized that duration and length measured in one frame of reference can differ from the same measurements made in a different frame of reference. But nothing about the object being measured has changed!
  - C. The mathematical description for trading time and space measured by one observer with time and space as measured by another observer is given by a set of formulas called the *Lorentz transformations*.
  - D. In string theory, there is the idea of extra dimensions, but there are no experiments yet that *demand* the scientific belief that these extra dimensions actually exist.
  - E. In 1999, two physicists, Randall and Sundrum, proposed a view that our universe may be described in a larger world as if it were a pane of glass. This is called the *brane-world scenario*. With acceptance of brane-world scenarios, certain things in physics become much more obvious.
    1. In our world, the gravitational force is the weakest of the forces that act on matter. Why is gravity so weak?
    2. If our world is like a pane of glass, perhaps some gravity is “leaking” into other dimensions, thus diminishing its effects.
  - F. Though widely accepted by the majority of scientists who consider such matters at present, the *brane-world scenario* cannot be settled until there is some sort of experimental evidence, either direct or indirect.
- II. The idea of “extra” dimensions did not start in string theory.
- A. In the 1930s, Kaluza proposed that the physics of our world required at least one extra dimension. When Einstein first read the paper, he thought it was nonsense, but on later reflection, he found it very interesting to consider: Maxwell’s equations could be found in the mathematical results combining Einstein’s own theory of general relativity with the idea of the extra dimension.
  - B. Klein pointed out that if this extra dimension were very small, it would be easy to overlook it.
    1. Imagine a house with many rooms, but one special room possessing a doorway so small no one could enter the room. If

one were able to pass through the doorway, it would be like any other room; it's the door that's the problem.

2. A way to avoid the problems of extra dimensions is to think about them as being very small or possessing very small "doorways."

C. In 1978, a group of physicists studying supergravity realized that the equations for the most complicated version of this theory could be simplified by the introduction of 11 dimensions. Thus, when string theory had its breakthrough in the 1980s, physicists were already thinking about extra dimensions because of supergravity.

D. Are extra dimensions real? Present-day technology is limited in its ability to answer this question. This situation often occurs in physics.

III. The idea of extra dimensions has also shown up in another way in physics.

A. Earlier, it was thought that the smallest electrical charge was the size of that charge carried by the electron. With the acceptance of the existence of quarks, evidence showed that electrical charge could come in fractions of  $1/3$  or  $2/3$  the charge of an electron.

B. Charge still appears to occur only in "chunks," but these chunks are of a smaller size than originally thought. Physicists have long wondered why this is so. One of the ways to answer this question was proposed by Kemmer in 1938 in his explorations of protons and neutrons.

1. Accounting for spin, electrons seem to come in only two varieties; they either spin up or spin down. That is similar to electrical charge. Kemmer tried to apply the mathematics of spin to charge.
2. Protons are charged; neutrons are neutral; and the two are almost the same in terms of their mass. Kemmer posited that, on some deeper level of reality, protons and neutrons had the same mass. If that were true, then it is only the charge that distinguishes them.
3. He further posited that charge was the same as spin. If that were true, protons and neutrons would not be different objects but the same object "spinning" in different ways. The implications of the mathematics of this hypothesis, known as *isotopic charge space*, work perfectly well to describe the

physics of the denizens. The mathematics of spinning objects in our world can be applied to things that possess electrical charge.

4. Kemmer's notion of charge also includes mathematical objects that behave like angles, but they are not dimensions to which we have access. In fact, his angles are precisely the ones associated with the force carriers. Ordinary angles describe different orientations of objects in space. So the occurrence of large numbers of angles would seem to imply lots of different directions for space.

C. Throughout, the standard model uses the idea of mathematical quantities with the properties of angles. Recall the fact that changes in two different photons can produce the same electrical field. Two photons that possess this property have mathematical descriptions that are related to one another by an angle.

D. Even for physicists who do not accept the belief in extra dimensions—and many physicists don't—the mathematics that describes the electromagnetic force behaves in many ways as if it has three spatial dimensions, one temporal dimension, and one other dimension that allows for the angle described above.

IV. Now a return to one of the “dirty little secrets” of the heterotic string.

A. Recall diagrams representing the polarization of light—also related to descriptions of standing waves, right-moving waves, and left-moving waves in the heterotic string.

B. The standing-wave mode corresponds to one of the strands of the string that describes space; thus, we need nine of these standing-wave modes to describe the space of the heterotic string and one to describe time.

C. When the animation is in motion, we can see that superimposing the representations of the left- and right-movers actually yields only the standing wave. Can we reverse that process? Is it possible to mathematically dissect a standing-wave mode into its left- and right-movers?

D. Using a mathematical tool called *Fourier analysis*, we find that the standing wave is exactly equal to the mathematical expression for a left-mover added to the mathematical expression for a right-mover.

- E. This means that anything associated with a standing wave can be pulled apart mathematically into left-movers and right-movers. Why is that important? It turns out that the angular variables of string theory, which are associated with the different photons that produce the same electromagnetic force, are always either purely left-movers or purely right-movers, but never standing waves.
- F. This understanding gives string theory a kind of flexibility. What might be initially identified as a dimension (associated with standing modes) can actually be pulled apart and traded for more charge! The mathematical structure of the model can be reduced to less than 10 dimensions. Effectively, a trade-off of directions for charge occurs.
- G. This is a potent idea. In studying the mathematics of the heterotic string, there can be two interpretations. One (the presently most popular) interpretation argues for extra dimensions, but the alternative asserts that what can be misidentified as extra dimensions are actually angular variables associated with the changes of force-carrying particles.
- H. In string theory, what has traditionally been identified as a dimension may actually be two kinds of charges: left charge and right charge. In the original heterotic string, all 496 charges are associated with right-movers and the theory possessed nine spatial dimensions and one temporal dimension.
  - 1. It is possible to carry out an operation on one of those nine mathematical objects describing the space of the heterotic string and pull it apart into *left-charges* and *right-charges*. In fact, if this is done to six of these dimensions, the result is an object that exists in four dimensions (which looks a lot like our world).
  - 2. The mathematics of the heterotic string allows a construction where the number of dimensions is reduced to four, but in the process, it is necessary to introduce more than 496 right-charges.
- V. One might also wonder what is happening to the left-movers. For every extra dimension encountered, after being pulled apart to get a right piece, which is a charge, there remains a left piece. With what should it be identified?

- A. My colleague Warren Siegel and I pursued this question of mathematically pulling the heterotic string apart and found another formulation of the heterotic string that has the same 496 currents as in the original version and something else.
  - B. The discussion about the denizens of the quantum world included the fact that the electron possessed copies. One copy of the electron, called the *muon*, is 200 times as heavy, and a second, the *tau particle*, is 1700 times as heavy.
  - C. For a string theory in four dimensions, this work showed that the mathematics automatically produces copies of particles in the theory. Thus, a string theory written in four dimensions looks more like our world than a string theory written in higher dimensions.
  - D. Without string theory, there is no explanation for why copies of the electron exist. With string theory, we can see that the copies correspond to the absence of extra dimensions, which were pulled apart into charge and a property that is responsible for what is called *family number* (i.e., the copying) in the standard model.
- VI. Most of the popular discussions about string theory include the process of compactification.
- A. Previous discussions considered that extra dimensions might exist and be very small, but they might also be incredibly warped. If string theory allows for extra dimensions, it ought to explain why they must be small and warped. There is no complete and consistent explanation about this known to your lecturer.
  - B. On the other hand, if we start from the point of view that string theory ought to describe our world, then by definition, one ought to allow for string theory to naturally provide for the copying processes seen in the standard model, as well as the angular variables that are not associated with actual motion but are associated with objects like the photon. Taking this view, then, string theory comes to live comfortably in four dimensions.
  - C. In the four-dimensional point of view, there is no question about warping, because the theory has no options. If one believes in extra dimensions, then hidden questions arise: Why did the theory warp itself? Are all possible compactification techniques known?

- D. These questions will probably not be settled for decades or perhaps even centuries. Is string theory a four-dimensional construct, or is it a higher-dimensional construct that involves compactification?

VII. Let me make a few closing comments on four-dimensional string research.

- A. In the mid- to late 1980s, three groups of physicists studied the question of four-dimensional strings. One group extensively studied the idea that the heterotic string is an actual four-dimensional construct, using a mathematical approach called *free fermions*. Another group used a technique called the *covariant lattice*. The drawback with all these approaches is that strings are much more complicated to write mathematically as four-dimensional objects than their higher-dimensional analogs.
- B. In recent years, physicists have been looking at  $G2$  *compactifications*, *spin-bundle compactifications*, and *fluxes*. These are all different mathematical tools for imagining the warping of higher dimensions. But we have no guarantees yet that, with the higher-dimensional point of view, a complete determination of the right way to “bend” the extra dimensions to produce our world has been found.
- C. On the other hand, believing in the four-dimensional point of view requires the addition of many more “pieces” to our world that are not included in the standard model. The implications for this are not quite understood yet. If this problem is ever solved, it will likely become possible to write many versions of the string that contain our world and other “stuff.” What is this other “stuff”? Perhaps it is the dark matter discussed in Lecture One, copiously produced in a four-dimensional string.

### Readings:

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Halpern, *The Great Beyond: Higher Dimensions, Parallel Universes, and the Extraordinary Search for a Theory of Everything*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

Krauss, *Hiding in the Mirror: The Mysterious Allure of Extra Dimensions, from Plato to String Theory and Beyond*.

Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe*.

Randall, *Warped Passages: Unraveling the Mysteries of the Universe's Hidden Dimensions*.

Susskind, *The Cosmic Landscape: String Theory and the Illusion of Intelligent Design*.

**Questions to Consider:**

1. Describe the two interpretations of the heterotic string discussed in this lecture.
2. What problems are left in the higher-dimensional view of string theory?

# Lecture Seventeen

## The Fundamental Forces Strung Out

**Scope:** In a much earlier lecture, we discussed a dilemma encountered by Stephen Hawking with regard to black-hole radiation. In this lecture, we will consider how superstring theory apparently provides a rigorous mathematical proof to support Dr. Hawking's intuition. In the process, we will note that experimental support for string theory may come, not by looking at the smallest structures in the universe, but rather, by looking at the largest structure, the universe itself.

### Outline

- I. A prediction of general relativity (the theory describing how gravity works in our universe) was that objects might exist that are so massive that nothing can get away from their gravitational pull. These objects are black holes.
  - A. A black hole is not just a singularity but also has about it a sphere. The special property of the sphere is that light can orbit on it, just as satellites orbit around Earth. The sphere is called the *event horizon*.
  - B. General relativity also predicts that light can be bent by gravitational forces. This was shown in an earlier animation where starlight was distorted in the night sky as a black hole passed across a field of view.
  - C. The gravitational pull of a black hole is so strong that nothing can escape from inside the event horizon. In the late 1960s, Bekenstein took the first step in showing that black holes ought to radiate heat. In a brilliant insight, Hawking asserted that the usual arguments about black holes do not account for laws that apply in the world of the quantum.
    1. Our world is such that if two balls are thrown at each other, they will collide and bounce off one another. But two flashlight beams pass through each other, as if one doesn't sense that the other is there. These are classical expectations.

2. Using the laws of the quantum world, light and everything else behave differently.
3. To understand the quantum behavior of light, think of a Feynman diagram showing a box with two wavy lines, representing light, attached to the bottom vertices. According to  $E = mc^2$ , energy can be traded for mass; thus, the sides of the box represent the motion of a particle and an antiparticle (an electron and anti-electron). At the top vertices of the box, the photons stream away. This diagram describes a process called *Delbruck's scattering*, predicted in 1933. According to  $E = mc^2$ , energy can be traded for mass; thus, the photon can be replaced by a particle/antiparticle pair. The sides of the box represent the motion of this particle and antiparticle. At the top of the box, the particle/antiparticle pairs have recombined to form a photon, and the photons stream away.
4. In 1953, an experiment was performed that showed that light can scatter itself. Remember, the source of the scattering is quantum mechanical processes. This example shows us that to understand our world completely, it is sometimes necessary to take into account quantum processes.

**D.** In previous lectures, we discussed the idea that the force of gravity, as described by Einstein's equations, does not take into account the workings of the quantum world. Einstein's general relativity, combined with the laws of the quantum world, leads to infinite gravity, which would be a disaster. The definitive demonstration of this was given by a group of physicists in 1974.

**II.** The problem that Stephen Hawking encountered was that there was no mathematical way to support his intuition that hot black holes (like all other such objects in the universe) must radiate heat.

- A.** It is useful to review this crisis in physics in a slightly different way.
1. An animation shows why two electrons repel each other. The two balls of light represent electrons, and the wavy line between them represents the exchange of a message carrier (in this case, the photon), telling them to be repelled.
  2. In the quantum world, there are more complicated pictures. Again, the two spots of light represent electrons, but one of

them emits a photon, which it then later reabsorbs; it also emits a second photon that is responsible for the repulsion.

3. When Feynman Rules are applied to this picture, the mathematics that comes out implies that the force of repulsion between two electrons is not exactly what is taught in high school science.

**B.** This process can also be applied to the force of gravity. Using the same picture, think of one little spot as the Earth and one as the Moon. In this case, the wavy line used to represent the photon now represents the graviton, the message carrier of the gravitational force.

1. The graviton spins at 4 times the rate of an electron.
2. Applying Feynman Rules to the graviton, the result is infinity. There must be a way to change the mathematics.
3. For now, we have found only one other mathematical way to support Hawking's intuition about the ability of black holes to give off heat. Of course, this other model is string theory. The secret to supporting Hawking's intuition occurred with the photon that was emitted and later absorbed. If the emitted photon traveled outside of the black hole and the electron did not, the photon could be free to carry off the heat!

**III.** How is string theory different from previous constructions, and why does it seem to work?

- A.** An animation of two strings moving in space-time shows that they might join together to form a single string that continues moving as a unit. It is sometimes useful to consider the image of a y-shaped figure to show the same process. The two circles to the left of the open end of the y-shaped figure represent the two strings. The two strings move together until they touch each other, then join together and move off as a single string, represented by a single circle on the right side of the y-shaped figure. This y-shaped figure is called the *pants diagram*.
- B.** In the next animation, a single closed string divides itself into two strings. Again, it can be helpful to consider a still image, with the open end of the y pointing in the opposite direction. A string is indicated with the circle on the left side of the y-shaped figure; it then separates into two strings, represented by two circles on the right of the figure.

- C. A third animation can combine these processes. Two closed loops join together to become a single closed loop for a while; this loop then divides itself and becomes two loops again.
- D. Because strings are smaller than molecules of air, they don't produce sound when they vibrate; when strings vibrate, the only way for humans to detect them is as the particles introduced as the denizens of the quantum world. As the two strings come together in the animation, it is possible to consider a single vibrational mode of the string.
- E. This animation shows the path of vibrational modes as a v-shaped dotted line to the left and right of the wavy line. In the middle, when the string is a single unit, there is a wavy line, because when the strings join together, the modes of vibration change. If studying properties of the electromagnetic force, the carrier in the middle must be a photon. But if considering the force of gravity, the carrier in the middle must be a graviton. It is possible to look at this process as a still picture with the two y's connected in the middle.
- F. By applying Feynman Rules, calculations of mathematical expressions from these pictures can be obtained. Further, for every diagram possible for a string, a simple "pinching" of the string diagram yields the same process for particles. Remarkably, every possible way to "pinch" the string diagram leads to a well-defined mathematical expression for a force. It is not separate objects that transmit the forces, then; it is always the string itself. The pinching applies to all possible pictures and allows the "wanderer," discussed above, to carry away the heat of the black hole.
- G. As early as 1980, the physicist Daniel Friedan used this idea to calculate from string theory how the forces work. For physicists, this is a calculation called the *effective action*.
  1. The ordinary action gives us the effect of all the classical pictures—that is, our first diagram with no complications in the middle.
  2. As more complicated interior structures are added to the diagram, they must produce more complicated mathematical expressions. The effective action is what keeps track of all of these pictures. By having all the pictures built into it, a complete description of the force emerges.

3. Friedan showed that Einstein's equations can be derived from the equations of string theory. Further, general relativity is not required to carry out this derivation.
- IV. This idea of calculating quantum corrections can be applied to the problem of black-hole radiation.
- A. In 1996, two physicists at Harvard, Andrew Strominger and Cumrun Vafa, applied the picture-making technique of superstring theory to try to understand how black holes can produce heat. By using rigorous mathematical rules that follow from the structure of string theory, they were able to produce exactly the result that Hawking had found through intuition.
  - B. The lesson here is that string theory is the only piece of mathematics known that supports the idea that hot black holes can radiate heat.
- V. Many have claimed the existence of rivals to string theory. One of these rivals is called *loop quantum gravity*.
- A. Loop quantum gravity starts from a different viewpoint. The loops in this theory are not like the filaments of string theory but can be thought of as plaquettes, flat plates that are used to measure how space is bent.
  - B. Several years ago, a group of physicists used the formulas of loop quantum gravity to try to calculate Hawking-Bekenstein radiation. The initial results indicated that loop quantum gravity was capable of making this calculation. Within the last year or so, however, the calculations of loop quantum gravity have been found not to agree with Hawking-Bekenstein radiation.
- VI. What good is our quantum theory of gravity?
- A. This question cannot be answered yet. To take advantage of this theory in the realm of technology may require advances in technology of several hundred years.
  - B. What else does the quantum theory of gravity mean for our universe?
    1. In an early lecture, the cosmic microwave background was introduced. It is seen everywhere in the heavens and represents our universe when it was about 300,000 years old. It looks similar to static on an old TV set.

2. The average size of the “spots” in this static can be measured to get information that may be useful for studying string theory. Understanding the fine structure in the cosmic microwave background depends on the equations that describe gravity.
3. As seen in this lecture, the equations of string theory describe gravity in a way that is consistent with the laws of quantum mechanics. Ordinary general relativity does not do so. Thus, there must be attributes in the distribution of the spots of the cosmic microwave background that potentially encode this different description of gravity, if the physics of black holes are a hint.
4. It’s possible, then, that evidence of strings will not be found in the world’s most powerful accelerators. It may be necessary to look up at the heavens and try to read the signs of string theory in the structure of the cosmic microwave background.

### **Readings:**

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Hawking, *The Universe in a Nutshell* and *A Brief History of Time*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

Smolin, *Three Roads to Quantum Gravity*.

### **Questions to Consider:**

1. Describe the action of a photon carrying a message between two electrons in classical physics.
2. What’s different about this picture in the quantum world?

## Lecture Eighteen

### Do-See-Do and Swing Your Superpartner—Part I

**Scope:** Prior to string theory, space and time were separate from the forces and matter that exist in the universe. String theory gives us a different viewpoint, in which space and time are actually part of everything else and vice versa. This lecture will concentrate on the other “stuff” in the universe, namely, the parts that lead to creatures like humans. Mathematical evidence pointing to the existence of *superpartners* for this ordinary matter will be found, and we will also confront the fact that these superpartners have yet to be seen in the laboratory.

#### Outline

- I. Our universe contains objects, the things that make up matter, such as quarks and leptons. The other part of the universe that leads to creatures like humans consists of the forces, which hold the fundamental building blocks of matter in fixed patterns.
  - A. A neutron, when separated from the nucleus of an atom, on average, after 1013 seconds, is replaced by a proton, an electron, and a neutrino. The initial neutron was neutral; the proton present afterwards is positively charged; and the electron is negatively charged. This process is called *beta decay*, and the form of energy responsible for beta decay is the weak nuclear force. Other forces include electromagnetism, very familiar to many people, and the strong nuclear force.
  - B. All the matter objects we have discussed have the same spin rate as the electron. They can spin clockwise or counterclockwise at a rate of  $1/2 \hbar$ .
  - C. The force carriers have spin rates that are very different. The graviton spins at 4 times the rate of the electron. The photon, the intermediate vector bosons, and the gluon all spin at 2 times the rate of the electron. The Higgs particle doesn't spin at all.
  - D. Spin indicates which particles obey the laws of chemistry and which don't. Electrons gather around atoms in shell structures that are governed by spin. Since quarks have the same spin as

electrons, their shell structure would have to look very much like an atom. In contrast, trying to build an atom with photons, the shell structure would disappear, because anything that spins at 2 or a multiple of 2 times the rate of the electron does not obey the Pauli exclusion principle.

- II.** Why is it that the forces don't obey the Pauli exclusion principle and can never lead to a shell structure, whereas all the matter fields do obey the Pauli exclusion principle and will always lead to something like a shell structure for atoms? We might think our universe would be more balanced if some of the force carriers obeyed the Pauli exclusion principle and others did not and if matter particles behaved in the same way.
- A.** It is useful to look at a graphic indicating the parts of our universe: the quarks (red), the leptons (blue), and the force carriers (green). In an earlier lecture, we introduced the property of supersymmetry to banish the tachyon from the mathematical structure of the string. In supersymmetry, for every bosonic strand of the string, there had to be a corresponding fermionic strand of the string.
- B.** Is our world supersymmetric? We can find out simply by counting objects in the standard model, beginning with 18 quarks (6 flavors  $\times$  3 colors) + 6 leptons = 24 fermions. In fact, each fermion counts as 4 objects, accounting for the spin-up and spin-down varieties and for the fact that each fermion must also account for a particle and an antiparticle. Thus, the number of fermionic degrees of freedom in our universe is  $24 \times 4$ , or 96.
1. Next, it is possible to count the force-carrying particles: 8 gluons + 1 photon + 3 intermediate vector bosons + 1 Higgs. Again, the counting is a little bit more complicated. The photon counts for 2 states, because it can spin to the right or to the left like an electron; thus, the photon has 2 degrees of freedom. This is also true of the gluons. Intermediate vector bosons have 3 degrees of freedom. Adding them all up, we get 29 bosonic degrees of freedom. (For now, we ignore the Higgs boson since no experiment has yet seen it.)
  2. Our universe has no hope of being supersymmetric, because 96 does not equal 29 (and including the Higgs particle does not help).
- C.** This result presents a dilemma that involves the vacuum.

1. The vacuum is a state of lowest energy. Accounting for quantum processes, the concept of the vacuum in the world of the quantum is not as simple as might be imagined.
2. A common concept is that there is nothing in a vacuum. However, in a quantum mechanical universe, particles can spontaneously come into existence if they come as particle/antiparticle pairs. Rather than thinking about the vacuum as a state of nothingness, it must be accepted, from the principles of quantum mechanics, as a sea in which particles pop in and out of existence.
3. Thus, in quantum theory, the vacuum has a much more sophisticated structure. This raises a question of how to calculate the energy of a vacuum. Using the mathematics of quantum theory, this is a very difficult question...with one exception. It turns out that if the universe is truly supersymmetrical, then the vacuum can be shown to be a state of lowest energy.

**III.** This presents a dilemma. By counting, the universe is seen to be not supersymmetrical, but the vacuum would necessarily be the state of lowest energy if it were.

- A. The weak nuclear force is the form of nuclear energy that turns a neutron into a proton. When this process was first discovered, available technology was such that measurements of the neutron, the proton, and the electron—but not neutrinos—were possible. Laboratory results showed that the energy after the decay did not equal the energy before. This was also true for momentum. If this result were true, then energy and mass are not the same thing, which contradicts  $E = mc^2$ .
- B. Wolfgang Pauli asserted the reason energy and momentum appeared not to be conserved in this process is that something was escaping the detectors. This “something” is now known as the neutrino.
- C. The lesson here is that sometimes it is possible to imagine a slightly different universe mathematically, but later, experiments must support this different universe.

- IV.** It is possible to imagine a supersymmetric universe.
- A.** In a supersymmetric universe, the matter and force carriers known to exist thus far must have supersymmetric partners. In our universe, if we have, for example, the graviton, which has a spin rate of 2, then in a supersymmetric universe, there must also be an object that carries the gravitational force but spins at a rate of  $3/2$ . Such objects have never been seen in the lab.
  - B.** Further, in our world, the electron exists, which spins at a rate of  $1/2$ . In a more balanced world that is supersymmetric, there must be an object that spins at a rate of 0 (the *selectron*). Up and down quarks also spin at a rate of  $1/2$  in the supersymmetric world; up and down *squarks* spin at a rate of 0.
  - C.** In the graphic there is a balance on both sides. In a supersymmetric universe, for every matter particle and message carrier known, there is a superpartner.
  - D.** Of course, this whole idea is mathematics; it is possible to describe objects never before seen, but if they exist, calculations show they would serve to stabilize the vacuum.
- V.** Energy is a conserved quantity, and it is expected that there is a state when it is at its lowest value, the vacuum. How do those facts relate to the absence of supersymmetrical particles?
- A.** To answer this question, we might say that the supposition that the universe is supersymmetrical is incorrect. In string theory, the property of supersymmetry was needed to banish the tachyon.
  - B.** Supersymmetry at the level of the string asserts that the number of strands of the bosonic variety should be equal to the number of strands of the fermionic variety. But the particles in our world are not the actual strands of the strings; the particles are the objects that correspond to the notes produced by the strands. It is conceivable that even though the strands are supersymmetrical, the notes do not come in pairs. A few mathematical string models have this property, but most of the mathematical constructs that come from string theory have both the strands and the notes appearing in pairs. Thus, most physicists think that we ought to be able to find the superpartners.

- VI.** Why have superpartners not been found?
- A.** To see these superpartners, technology that is capable of producing them is required. When this will occur will depend on how massive these objects are.
  - B.** Typically, the mass of the superpartners is estimated to be 300–1000 times greater than the mass of ordinary matter.
  - C.** How fast is technology advancing? Consider the plot of the masses of the particles versus the history of detecting particles. The resulting straight-line plot looks like Moore’s Law, which states that the power of computers doubles once every two or three years.
  - D.** This same law can be applied to our ability to detect increasingly massive particles. We progress at a rate of about 3 GeV (giga-electron-volts—a unit of energy) every year. If superpartners are 300 times as heavy as ordinary matter, then we might produce one in a laboratory around the year 2016. But if they are 1000 times heavier than ordinary matter, producing one in the laboratory will take 200–300 years.
  - E.** The supersymmetric structure also depends on rigorously supporting experimental physics.
    - 1.** In our nation, about 10 years ago, the community of particle physicists had plans to build the Superconducting Super Collider. Ultimately, it was decided that this device was too expensive for the scientific budget of our nation to support.
    - 2.** Europeans have invested in a device about one-third as powerful as the Superconducting Super Collider would have been. This is the Large Hadron Collider (LHC), which is scheduled to be completed in 2007.
    - 3.** The United States has spent approximately \$500 million to put detecting devices in the LHC. In this way, Americans will participate in the search for what lies beyond the standard model, and many physicists believe that supersymmetry will be found in the LHC.
  - F.** In 1980, while visiting the University of California at Berkeley, I met Bruno Zumino, one of the physicists who first introduced the idea of supersymmetry in the Western literature. This was also the same year in which the mathematical idea of supersymmetry was applied to the standard model.

1. Zumino, one of the fathers of supersymmetry, was not as excited about this prospect as might be expected. Sometimes, ideas in physics turn out to be correct, but not in the way we think they are.
2. In the standard model, there's a special set of equations to describe gluons and intermediate vector bosons. These are the Yang-Mills equations, named after the two physicists who wrote them in 1954. Initially, when these equations were presented, they were created to understand forms of matter related to protons and neutrons. The equations failed miserably at this task, but 20 years later, they worked remarkably well for the weak intermediate vector bosons. About 10 years later, the intermediate vector bosons were found in the laboratory.
3. It is possible that supersymmetry might follow a similar route. Even though many who have studied supersymmetry find the mathematics to be elegant and convincing, until supersymmetry is observed in the lab, it will not be known if the mathematics describes our world. It is possible this mathematics is being applied incorrectly, but I believe that, if so, these equations will return, maybe 100 years from now, and describe something else of great importance.

### **Readings:**

Gell-Mann, *The Quark and the Jaguar: Adventures in the Simple and the Complex*.

Lederman, *The God Particle*.

Lederman and Hill, *Symmetry and the Beautiful Universe*.

Oerter, *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*.

Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics*.

### **Questions to Consider:**

1. When we counted the degrees of fermionic and bosonic freedom in our universe, what did this tell us about supersymmetry?
2. Why have we not yet seen any of the superpartners?

## Lecture Nineteen

### Do-See-Do and Swing Your Superpartner—Part II

**Scope:** At the end of the last lecture, we saw the possibility that the present understanding of the particle physics population could double. We considered a graphic of a balanced world, in which every matter particle seen in the laboratory has a superpartner yet to be seen. Ideally, seeing these superpartners in the laboratory would be preferable, but it is much more likely that an indirect method will yield the first experimental signatures. This lecture introduces this slightly different way to find the superpartners: through the unification of forces described by the running of the coupling constants.

### Outline

- I. A familiar statement from high school science states: The formula for the force between two charged objects is the product of the charges divided by the square of the distance.
  - A. This equation also comes from Feynman's picture-making method of understanding physics. A picture seen many times in this course shows a photon traveling between two electrons. There is an interesting feature about the mathematics of this picture. Each one of the factors of charge in the standard formula for force law is associated with the points in the diagram at which the electrons and the photon touch. The value for this mathematical feature is called the *coupling constant*.
  - B. What would happen in the picture if the charge of the electron were set to 0? The wavy line in the middle of our diagram, representing the photon, no longer touches the v-shaped lines representing the paths of the electrons. If that were the case, there would be no force, and this can be seen in this mathematical formula:  $F = \frac{\text{charge}^2}{\text{distance}^2}$ ; if charge is set to 0, then there is no force. A coupling constant controls how strongly the forces repel or attract.

- C. In a famous story, Archimedes was sitting in a bathtub one day and had a dazzling thought, causing him to run out of his house and yell, “Eureka!” He had figured out why objects float. His discovery is called *Archimedes’s principle*: When an object is placed in water, the amount of force with which the water pushes up is equal to the acceleration of gravity times the density of the water times the volume of water displaced when the object is pushed underwater.
- D. A swim in the Great Salt Lake of Utah will show that a person floats at a higher level in that water than would occur in the ocean, because the salt in the lake causes the lake water density to be greater than the seawater density. Archimedes’s law says that the buoyant force is proportional to the density. As the density of water increases, the upward force increases. If the density is set to zero, there is no buoyant force. That is like setting the coupling constant to zero in the Feynman diagram.
- E. In 1953, two physicists, Ernst Stückelberg and André Petermann, found that the electric coupling constant in the quantum world acts exactly like the density of water in Archimedes’s principle. The strength with which two objects in the quantum world repel each other depends on the energy at which the observation is made! The more complicated pictures of the quantum world, when added to the classical picture, change the way in which the photon communicates with a charged particle. With higher energy of the photon or the electrons, the coupling constant changes. Energy is, in the quantum world, as salt is in the water for buoyancy.
- F. This is true not just for the electromagnetic force but for all four forces, each of which is characterized by a coupling constant. For the force of gravity, that coupling constant is Newton’s constant  $G$ ; for the weak force, the constant is  $g_w$ ; for the strong force, it is  $g_s$ . Their role is the same as the coupling constant for the electromagnetic force.
- II. In the diagram showing the exchange of a message carrier, the electrons may be replaced with quarks. How do quarks interact in this scenario?
- A. Quarks interact by exchanging gluons, which keep the quarks permanently bound in the interiors of protons and neutrons. Since the force caused by the gluons has a coupling constant, we can

again ask what happens mathematically when the coupling constant is set to zero. As in the case of electromagnetism, the exterior lines in the diagram representing quarks no longer send message carriers back and forth, and without the message carriers, there are no forces.

- B. The same is true for gravity. If we replace the electrons in the original Feynman diagram with the Earth and the Moon, the exchange of gravitons is responsible for the attraction between the two. If Newton's constant is set to zero, then the graviton does not carry the message from the Earth to the Moon, and there is no attraction. This is a universal way of understanding how forces arise in the quantum world.
- C. The world of Stückelberg and Petermann implies that the coupling constants of nature aren't constant, a surprising result, but a quantum mechanical one—not something that affects us in everyday life. If the charge of electrons in the everyday world were to change arbitrarily, electricity or means of communication would be unpredictable.

III. In 1974, a second chapter was added to this story. In that year, three physicists, Howard Georgi, Helen Quinn, and Steven Weinberg, showed that the rates at which the coupling constants change can be calculated. Their equations describe what is called the *running of the coupling constants*.

- A. Plots of running coupling constants can be made on a graph. One axis is the energy axis, labeled  $E^2$ , and the other axis measures the strength of the coupling constants:  $G$  for gravity,  $E$  for the electromagnetic force,  $g_s$  for the strong force, and  $g_w$  for the weak force. Four points in the graph represent the four forces; gravity is the weakest force, followed by electromagnetism, the weak nuclear force, and the strong nuclear force.
- B. The speed to which particles are accelerated determines the energies at which an experiment is performed. Higher speeds imply higher energies. We can illustrate this point by imagining colliding two beams of electrons.
  - 1. The force of repulsion can be measured while the electrons are moving at a relatively small velocity. The same measurement can be made for increased velocities.

2. Remarkably enough, results of such experiments in the quantum world show that the coupling constants do change, as seen in the plot: The strong coupling constant is a bit smaller, the weak coupling constant is about the same, and the electromagnetic and gravitational coupling constants have both increased.
- C. What happens at even higher energies? The coupling constants move even closer to each other. Gravity gets even stronger, the electromagnetic interaction stays about the same, the weak interaction gets a little bit stronger, and the strong interaction decreases. At even higher energies, the electromagnetic and weak coupling constants merge. This energy is then called the *electro-weak unification energy*. This change in the coupling constants is actually something that can be seen in a laboratory experiment.
  - D. At even higher energies, the strong nuclear force, weak nuclear force, and electromagnetic forces all join and have the same strength. The only outlier is the force of gravity. Of course, at an even higher energy, gravity will join the other three forces. This energy is very close to what is called the *Planck energy*, and at this level, gravity and the other forces are indistinguishable.
- IV. At this point in the presentation, the discussion has shown how coupling constants *run* in the microcosm. This has interesting implications for the standard model.
- A. In a second animated plot of the forces, we see the paths that the coupling constants for each force follow as they become unified. The paths shown can be calculated using the method of Georgi, Quinn, and Weinberg.
  - B. Notice that the electro-weak unification occurred before the strong force joined. The coupling constant for gravity unifies with the others even later. It is interesting to reconsider this pattern of unification but in the context of a supersymmetric version of the standard model.
  - C. If the particles in the standard model have superpartners, that has an impact on the running of the coupling constants. Considering a Feynman diagram of the electromagnetic force, the superpartners also could play a role, via loops, of modifying the effects of the standard force carriers. Since superpartners have slightly different properties from ordinary matter, they will change the way that the

coupling constant runs in a slightly different way. This makes it possible to ask: If our world is supersymmetric, how do the coupling constants run?

- D.** The physicist who first performed these calculations, in 1984, was a Russian named Dmitri Kazakov. His work showed that, in the supersymmetrical model, the strong, weak, and electromagnetic forces all unify at the same energy. That's a different behavior than seen in the standard model without superpartners.
- E.** Physics is often driven by a sense of elegance. In this case, many physicists have argued that the supersymmetrical picture of the unification of forces is a more elegant result than the standard model. Unfortunately, our sense of beauty does not determine physics. Physics is determined by laboratory experiments, but this understanding of the unification of forces offers another window through which to determine whether our world is supersymmetric.
- F.** In the last lecture, we saw that, depending on the masses of the superpartners, we may not be able to see them for hundreds of years. The coupling constants offer a different path to discovery. The ability to measure the coupling constants does not depend on first seeing the superpartners. Instead, it relies on careful measurement of ordinary forces between ordinary matter. Another way to detect the presence of superpartners, then, is to measure the paths of the coupling constants.
- G.** If experiments at the LHC indicate that the paths of the coupling constants are the ones we associate with the standard model, this will be damning evidence against the existence of supersymmetry. If the paths of the coupling constants point to a single unification of the strong, weak, and electromagnetic forces, that will be pretty strong evidence that ours is a supersymmetrical universe, even though superpartners are not yet seen.

**V.** Why should the existence of superpartners matter?

- A.** The concept of an electron began as an idea. Of course, now, a great deal of present-day technology is based on this idea. This provides a good example of what can happen when humanity discovers new forms of matter and energy. There is a long-term potential for new and unimagined forms of technology.

- B.** What might these technologies be? Science fiction permits speculation. Perhaps it is possible to transmit ordinary electrons into selectrons. We have seen that, if it were possible to turn off the charges for ordinary electrons, they would still repel each other. But selectrons are bosons; if their charges are turned off, they can pass right through each other.
1. In present-day experiments, a process called *quantum teleportation* is being studied in laboratories. This process is possible because photons are bosons, so many can be superimposed at the same place at the same time.
  2. In turn, this leads to *entanglement*, which is also being explored for the possibility to create quantum computers. Since selectrons are bosons, it should be possible to entangle them. To turn off their electric charges would mean something akin to their further transmutation into electron sneutrinos. If these were to accelerate to near the speed of light, their lifetimes would increase while they would retain the attributes of quantum entanglement.
  3. Further, if all this were to be a reversible process, the end result would be a matter transportation process, right out of *Star Trek!* While this is all simple and wild speculation, it is interesting to imagine how the discovery of supersymmetry in our universe could be the key to creating transporters.

### Readings:

Gell-Mann, *The Quark and the Jaguar: Adventures in the Simple and the Complex*.

Lederman, *The God Particle*.

Lederman and Hill, *Symmetry and the Beautiful Universe*.

Kane, *Supersymmetry: Unraveling the Ultimate Laws of Nature*.

Oerter, *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*.

Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics*.

### Questions to Consider:

1. Describe the running of the coupling constant in the standard model.
2. Describe the running of the coupling constant in the quantum world.

## Lecture Twenty

### A Superpartner for Dr. Einstein's Graviton

**Scope:** In this lecture, we discover that most of the superparticles will be enormously massive, but one of them, the *lightest supersymmetrical particle* (LSP), will be the least massive. Because this particle is supersymmetrical, it may not function according to the same force laws that act on ordinary matter. In that case, it will be dark and may offer an explanation for the dark matter problem in the structure of galaxy formation.

#### Outline

- I. Research conducted by your professor posited an alternative model for superpartners, but the focus here will be on the version of supersymmetry that has the widest support today.
- II. Can we find a mathematically consistent way to introduce mass to the superpartners so they become very heavy while ordinary matter remains very light?
  - A. Ordinary matter is paired with supermatter in supersymmetry. If this theory is correct, then the graviton must have a superpartner.
    1. Light can be left-circularly polarized or right-circularly polarized. Physicists refer to these as the degrees of freedom of the photon. This statement is also true for gravitons. The number of degrees of freedom for gravitons, like all force carriers, depends on the number of dimensions being considered.
    2. The formula to find the number of degrees of freedom of a graviton is:  $\frac{1}{2} D (D - 3)$ , in which  $D$  stands for the number of dimensions. For our world, this number is 4. If this is substituted in the formula, the result is 2.
    3. If  $D = 3$  in this formula, there are no degrees of freedom for gravity. Thus, if the world were three dimensional (2 spatial and 1 temporal dimension), there would be no gravity, and if there were no gravity, no galaxies, no planets, no stars, no life. The smallest conceivable mathematical construct that results in a force of gravity with degrees of freedom is 4...looks remarkably like our world.

- B. Einstein gave us a picture of gravity associated with curvature or acceleration. If a person were placed in an elevator, far from any planet, accelerating upward, all objects, such as a ball, would appear to fall. Even a beam of light would appear to fall because gravity truly does curve light. The message carrier responsible for this is the graviton.
- C. For supersymmetry to exist, exchanges of ordinary quanta of matter and energy must be made with superpartners. The object exchanged with the graviton must differ from the graviton by  $1/2$  unit of spin; it is called the *gravitino*.
- D. In the mid-1970s, the equations describing this object interacting consistently with gravity were first written by two groups, Ferrara, Freedman, and van Nieuwenhuizen, and Deser and Zumino. This theory of gravity with a superpartner is called *supergravity*.
- E. Supergravity has the same kind of angles associated with it as the photons. Earlier, we said that if the change in two different photons is the same, the result for each is the same electromagnetic field. This statement is also true of gravitons. If the change in two different gravitons is the same, the same gravitational effect is produced.

III. It is thus possible to have a general picture of supergravity, but our world is not supersymmetrical. No doubling of the particles has yet been observed.

- A. If superpartners are very massive, they have yet to be seen in experiments. Still, is it possible to find mathematical equations which allow superpartners to be much more massive than the quanta of ordinary matter and energy?
- B. Of the known force carriers, only those associated with the weak nuclear force have mass. The other force carriers are massless. This problem was not completely worked out until particle physicists borrowed an idea from physicists who studied magnets.
  1. Mathematical equations can be written to describe a phenomenon called *spontaneous magnetization*, where materials that are not magnets at normal temperatures become magnets if cooled sufficiently. These same equations were borrowed by particle physicists to study how to include mass for the force carriers.

2. For years, it was known that the most obvious way to include mass for force carriers results in time-like particles. Mathematics used to describe magnets came to the rescue. Brout, Englert, Guralnik, Hagen, Higgs, and Kibble applied equations borrowed from the description of spontaneous magnetization to particle physics, the result being the *Higgs mechanism*, which provides a way to give mass to ordinary force carriers without introducing time-like particles.

#### IV. A look at how the Higgs mechanism works is in order.

- A. It is believed that a spin-0 particle exists in nature called the *Goldstone particle*, whose energy takes the form of a Mexican hat. Our animation shows a ball, representing the Goldstone boson or bosons, on top of a hill. Below the Mexican hat, the animation has three dots, representing ordinary force carriers, such as the intermediate vector bosons.
  1. When the Higgs boson and the Goldstone fields are at the top of this energy potential, the ordinary force carriers are massless. As the Goldstone and Higgs bosons together roll down this potential, the force carriers grow massive.
  2. In the picture, one of the force carriers, the photon, did not grow mass.
  3. In a different animation, we see that when the Goldstone boson reaches the bottom of the energy potential, it leaves another energy field related to the Higgs boson throughout space and time.
  4. The disks in the second animation represent the remaining energy of the Higgs boson pervading space. As an ordinary particle that would be massless moves through this field, it grows mass.
  5. This seemingly inelegant method is the only known way to provide mass to all matter and force carriers that occur in our universe without introducing other problems like time-like particles.
- B. This is how physicists believe mass is given to the ordinary force carriers as well as matter fermions, but remember, there is the other problem of giving mass to the superpartners.
  1. If supersymmetry is a feature of this universe, then the force carrier for gravity must have a superpartner. This superpartner

has been identified as the gravitino, which spins at a rate of  $3/2$ . Initial attempts to write equations for this object also indicate an expectation for it to be massless.

2. Gravity couples to everything, both ordinary matter and supermatter. This means that the superpartner of gravity must also couple to ordinary matter and supermatter. This cannot be all there is to this story. If it were, long ago it should have been possible to detect the effects of a massless gravitino.
- C. When bosons acquire mass through the Higgs mechanism, the kinds of angles associated with change in a force carrier more or less disappear, and when these angles disappear, some symmetries are apparently lost. This process is called *spontaneous symmetry breaking*.
- D. In the early 1980s, physicists wrote a model in which, initially, everything is massless. To include the effects of gravity and its superpartner, the model repeated for supersymmetry exactly the process that was carried out to give mass to the ordinary force carriers. In other words, the goal was to find a supersymmetrical version of the spontaneous symmetry-breaking mechanism.
- E. What are the implications of this model?
1. In the ordinary Higgs mechanism, as the Goldstone bosons roll down the energy hill, other particles gain mass. It is necessary to repeat that process in the context of supersymmetry.
  2. In supersymmetry, there should be an expectation to find a super-partner of the Goldstone particle sitting at the top of an energy hill called a superpotential. If the ordinary particle that sits at the top of a potential is spin 0, its superpartner would have spin  $1/2$ . This supersymmetrical partner has been named the *Goldstino*.
  3. In the supersymmetrical picture of the Higgs mechanism, there is both ordinary matter and supermatter. Also at the top of the hill picture is the Goldstino. As the Goldstino rolls down the Mexican hat, the ordinary particles remain massless (their masses are induced by the ordinary Higgs boson), but their superpartners all become much more massive, including the gravitino.
  4. Thus, by simply writing the equations in which gravity has a superpartner, then repeating exactly the same process that

gives mass to the force carriers and matter of our world, all superpartners can be shown to grow incredibly larger masses.

5. This mechanism is now called the *spontaneous breaking of supersymmetry*, and the implication of it is that supermatter is always much, much heavier than ordinary matter.
- F. This model solves another problem. As supermatter gets heavier, the superpartner to the graviton grows an incredible mass. This needs to be compatible with observation of our universe as all present-day experiments and observations point to one massless particle that is associated with communicating gravitational forces. If the supersymmetric Higgs mechanism had not worked, then in addition to the graviton, our universe would require a second object, the *gravitino*, responsible for transmitting gravitational forces over long ranges.
- V. With this picture, many of the previously discussed problems of supersymmetry disappear. We have never seen superpartners because they are so massive. The Goldstino, which is the superpartner to the graviton, is naturally a very massive object, and when it disappears from the theory, it leaves behind an energy field that gives mass to everything not yet seen with present-day instruments.

### Readings:

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

Kane, *Supersymmetry: Unraveling the Ultimate Laws of Nature*.

### Questions to Consider:

1. What aspects of our world might supersymmetry help explain?
2. How does the Higgs mechanism work in our universe and how can it be applied to a supersymmetrical universe?

## Lecture Twenty-One

### Can 4D Forces (without Gravity) Love Strings?

**Scope:** String theory did not begin as a project to describe the universe. In the early days of string theory, around 1968, physicists were simply looking for a way to understand nuclear matter. What we are now finding is that string theory can help us describe a physical phenomenon, the running of the coupling constant, that we have not yet been able to calculate mathematically. In this lecture, we will look at this phenomenon, along with other new ideas in string theory, including the *KLT relations*, *anti-de Sitter space*, and *conformal field theory* (AdS-CFT), the *brane world scenario*, and *NSR theory*, that may offer answers to real-world questions about the workings of nuclear matter.

#### Outline

- I. The lecture starts with a representation of a proton, consisting of a bubble and, on the inside, two up quarks and a down quark.
  - A. Nuclear matter comes in a second form, in objects called mesons. A representation of a meson has a quark and an anti-quark in its interior.
  - B. In 1968, physicists were struggling to determine how nuclear matter was put together. One possible mathematical description was called the *dual resonance model*, presented by Veneziano. The competing model was the *relativistic constituent model*, which ultimately, was judged to be the more accurate model.
  - C. The dual resonance model went on to become string theory, which can potentially be used as a way to understand the entire universe.
- II. As mentioned, strings come in two varieties, open and closed.
  - A. Among the closed strings, there is always a mode, or a manner of vibrating, that has the properties of the particle that carries the force of gravity, the graviton. The open string always has a mode that has the properties of a photon-like force carrier.
  - B. Strings seem naturally to exist in higher dimensions. When open strings were first studied, physicists thought that they must describe something similar to Maxwell's equations but in 9 spatial

dimensions and 1 temporal dimension (for the superstrings) or 25 spatial dimensions and 1 temporal dimension (for the bosonic string).

- C. To understand completely the mathematics of such objects, we must be able to count the number of degrees of freedom. The number of degrees of freedom for a 10-dimensional photon is different than that for a 4-dimensional photon; in general, the formula for determining the number of degrees of freedom is  $D - 8$  for photon-like objects.
- D. An open string carries charges at its ends, while a heterotic string allows charges to be distributed along the length of the string. In 1969, the only method for putting charge on the ends of the string relied on work of Chan and Paton; these charges are called *Chan-Paton factors*.
- E. The charges on the ends of the string are not limited to electrical charges. The weak nuclear charge or the strong nuclear charge can also be carried on the ends of the string. However, if the charge is something other than the electrical charge, then the particle that carries the charge must be the appropriate one. For example, if the charge at the ends of the string is color charge, then instead of a photon, the object that carries the charge must have properties like the gluon, which keeps quarks bound to the interior of nuclear matter.
- F. Much current research in string theory is devoted to understanding how protons and other nuclear particles work.

**III.** It is helpful to look at a mystery surrounding these particles.

- A. A return to the graph showing the strengths of the four force carriers is a convenient starting point. In looking at this plot in relation to variations with energies, we can see that the coupling constants move in opposite directions. For example, probing an object like the proton at smaller and smaller energies results in the force that couples the quarks together becoming stronger and stronger.
- B. In fact, in this version of the graph, the coupling constant for the strong reaction is shooting off into infinity. This phenomenon is required, as presently quarks are thought to be bound to the interior of particles, such as the proton or one of the mesons.

- C. Physicists do not know how actually to calculate this behavior of the coupling constant for the strong nuclear force in the described scenario, although there is a good deal of experimental evidence indicating the current understanding is correct. Using experimental probes at higher energies, this coupling constant is found to decrease.
- D. The question of why quarks can never escape the interior of hadronic matter reduces to understanding why the coupling constant for the strong force shoots off to infinity at smaller and smaller energies.

IV. In recent years, new ideas have come from string theory to try to understand this strange behavior.

- A. Part of this work comes from three physicists working in 1986, Kawai, Lewellen, and Tye, who observed that there must be some very deep, hidden mathematical relationship between a theory of gravity, provided by closed strings, and a theory that includes left-movers and right-movers, which are equivalent to open strings.
- B. Earlier animations showed two strings, one only supporting vibrations that moved in a left-hand sense, and one only supporting vibrations moving in a right-hand sense. Each of these has something to do with photons, gluons, or other force carriers in the standard model but not the graviton. This mysterious connection is now called the *KLT relations*.
- C. Remember the original problem: Why are quarks the only particles whose force carriers have a coupling constant that runs off into infinity at smaller and smaller energies?
  1. Think of a hadron as a bubble with quarks inside, and imagine that you could grab that bubble and start to pull on it.
  2. Using Feynman diagrams, the forces between particles are always carried back and forth by message particles, or *gauge particles*. For the strong force, these are the gluons.
  3. As the hadron is stretched apart, in our animation, gluons appear, moving back and forth between the quarks.
  4. Photons may fly around any path between a positive electrical charge and a negative electrical charge, but gluons cannot. They must remain packed tightly inside the bubble; when gluons are packed together, their energy starts to increase drastically.

5. Einstein taught that energy and mass are the same thing. As the quarks are moved farther apart, the gluons increase their energy to the point at which their energy can be converted into the mass of newly created quarks. In turn, new quarks allow the bubble to split; the result is two particles: one with a quark/anti-quark pair and the other with a three-quark combination, a baryon.
  6. No one has ever done a calculation to prove this process works as described above.
- D. The KLT relations give us an alternative way to study this process.
- V. In 1997, a totally new idea, AdS-CFT, was proposed. AdS stands for *anti-de Sitter space*; CFT stands for *conformal field theory*.
- A. In the 1920s, de Sitter was the first person to argue that our universe is one in which the curvature tensor is non-zero and positive. This situation is described as a de Sitter space or dS space. In anti-de Sitter space or AdS space, the curvature is non-zero and negative.
  - B. In a universe in which the only particles are those like gluons, the only sources of energies are those associated with gluons. However, if the mathematics of a theory that describes only gluons is given, it is found that the physics of the system always looks the same, no matter how deeply we look at it. This is called a *conformal system*. The field in CFT is the field of the gluons, and of course, T stands for theory.
  - C. AdS-CFT examines the relationship between a system that involves only gluons and relates it to a different theory about gravity with anti-de Sitter geometry, a geometry in which the curvature is negative.
  - D. This amazing discovery was made by Maldacena, a student of Witten. The discovery marked the beginning of the second string revolution. The first string revolution had occurred after Green and Schwarz discovered the magical number 496 that we talked about earlier.
- VI. To explain the shooting up of the coupling constant mathematically, physicists need to exploit new mathematical ideas. Some new mathematics has recently been proposed in the form of objects called *branes*.

- A. In an animation showing a small dot in the center of the screen, that dot may be considered to describe a charged particle, such as an electron. If it is an electron, it is free to move about.
  - B. Instead of calling this electron a dot, let's call it a *0-brane*. If it were a 1-brane, it would look like a line; in fact, it could be one of our strings. We could also construct a 2-brane, which would look like a two-dimensional plane.
  - C. In our world, the only other such object we could construct would be a 3-brane. In the mathematics of the string, however, nothing stops repeating this process all the way up to 10 as this corresponds to the 10 dimensions of string theory. The collection of all possible branes is called *Dp-branes*.
  - D. All the branes described must also be able to carry charge. Furthermore, they all must allow for force carriers that can couple to them.
  - E. Using Dp-branes has allowed us to find mathematical relations that imply a connection between a theory in which there are only gluons and a theory in which there is only gravity, and to derive an equation that connects gravity to a theory that involves only gluons. This discovery also enables physicists to calculate the coupling constant for the strong nuclear force at smaller energies.
- VII.** Witten has made an even more startling suggestion about how string theory can contribute to our understanding of the strong force.
- A. Recall the notion of anti-commuting numbers: The order of multiplication doesn't matter for ordinary numbers, for example,  $4 \times 3 = 3 \times 4$ . But in the 1800s, Grassmann studied numbers that had the property in which the order of multiplication does matter.
  - B. The anti-commuting property leads to a description of string theory called the *Neveu-Schwarz-Ramond (NSR) theory*. This theory includes strands that describe the string, as well as different mathematical strands that obey anti-commuting multiplication rules.
  - C. For every strand of the string, mirror copies can be introduced that possess the opposite multiplication law. One mirror copy leads to the  $N = 1$  NSR version of the string; two mirror copies lead to the  $N = 2$  version of the string. From the latter version, mathematical

objects emerge that are very similar to spinors, but these are called *twistors*, first proposed by Penrose in the 1960s.

- D. The notes of the  $N = 2$  string obey Einstein's hypothesis only if there are two time dimensions!

**VIII.** As we can see, a good bit of esoteric mathematics is being marshaled to understand nuclear particles.

- A. Although many people assert that string theory cannot be tested, this new horizon of string theory seems to be leading to mathematics that will allow the ability to calculate a physically interesting property, namely, the running of the coupling constant.
- B. Even though AdS-CFT has started to yield something that looks like real physics, it's not quite there yet. The only known way to test this idea is to work with something called a *type-IIB superstring*.
- C. In the next lecture, we will discuss all five of the superstrings. The IIB string was initially thought to describe only gravity and other denizens (but none like the gluons); with AdS-CFT, there is now evidence that gravity can be connected to the world of gluons. The mathematics of AdS-CFT implies that gluons can have four supersymmetrical partners, that is, four gluinos. This concept is called  *$N = 4$  supersymmetry* and is even harder to accept than the small concept of supersymmetry with only one partner for each particle in our world.

### **Readings:**

Gell-Mann, *The Quark and the Jaguar: Adventures in the Simple and the Complex*.

Lederman, *The God Particle*.

Lederman and Hill, *Symmetry and the Beautiful Universe*.

Oerter, *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*.

Zee, *Fearful Symmetry: The Search for Beauty in Modern Physics*.

**Questions to Consider:**

1. Describe the running of the coupling constant as it relates to the strong nuclear force.
2. What are some of the approaches that give physicists alternatives for calculating the coupling constant?

## Lecture Twenty-Two

### If You Knew SUSY

**Scope:** The expression *SUSY* means supersymmetric or supersymmetry. This lecture will lead to a rather unusual aspect of SUSY—*superspace*. Ordinary particles are free to move into superspace, but when they do, they become superpartners. The idea of superspace combines with the notion of strings to explain the existence of five types of superstrings.

### Outline

- I. What if there were more bizarre kinds of hidden dimensions?
  - A. In 1974, Salam and Strathdee invented a new kind of space that could accommodate our dimensions as one of its parts as well as these new bizarre dimensions. This construction is called *superspace*.
  - B. In our world, if we want to buy carpet for a room, we have to measure and multiply the dimensions of the room, but the order in which we measure the dimensions of the room doesn't matter. With algebra, it is possible to write an expression for this:  $x_1 \times x_2 = x_2 \times x_1$ . With this expression, we see that lengths are commuting numbers.
  - C. The work of Salam and Strathdee argues that, in order to describe physics, we may sometimes require numbers that correspond to the lengths that are commuting numbers, but they also investigated the possibility that some of the extra dimensions might not be numbers at all in the typical way numbers are used and envisioned.
- II. What prompted Salam and Strathdee to look into this idea was something that was already happening in string theory.
  - A. Salam and Strathdee were looking for a simple way to understand how bosons could be traded for fermions.
  - B. They hit upon an idea that involves extra dimensions. They argued that extra dimensions existed, but they used Grassmann numbers instead of ordinary numbers for these extra dimensions.

C. Salam and Strathdee introduced four of these extra dimensions, called  $\theta_1$  through  $\theta_4$ . If any two of these are multiplied, the result obeys Grassmann's anti-commuting rule. By these definitions, then,  $\theta_1 \times \theta_2 = -\theta_2 \times \theta_1$ . More remarkably, the square of any one of these numbers vanishes even though none of them is, itself, 0!

D. Why did Salam and Strathdee introduce four of these extra dimensions?

1. In the discussion about the electron, we stated that it has 4 degrees of freedom. It can spin in either a right-handed sense or a left-handed sense, and its antiparticle can also spin in either a right-handed or a left-handed sense.
2. One might be tempted to think that the antiparticle has nothing to do with the electron. But the mathematics that describes an electron and its antiparticle (the *positron*) can be mapped one onto the other if the antiparticle is interpreted as moving backward in time. This remarkable result was found by Richard Feynman.
3. This means antiparticles are not distinct from the particles they mirror; they are the same particles moving backward in time.
4. In that case, it makes sense to say that an electron spinning in either a right-handed or a left-handed sense is still an electron and a positron spinning in either a right-handed or a left-handed sense is also still an electron. Thus, an electron has 4 degrees of freedom.

E. Salam and Strathdee were not the first people to introduce anti-commuting numbers into physics. This honor belonged to a Russian physicist named Felix Berezin, who was more interested in practical problems in physics. Strangely, he found that introducing anti-commuting numbers actually simplified his calculations. Berezin is the father of what is now called *superanalysis*.

III. What is superspace? Superspace is a construct that has the coordinates of our world (duration, breadth, length, and thickness), along with four anti-commuting extra directions. This construct is called  $N = I$  *superspace*.

- A. More complicated versions of superspace are possible. We could imagine a mathematical construction that includes the coordinates of our world and adds two copies of these anti-commuting directions. This construct is called  $N = 2$  superspace. We might also imagine repeating this process to arrive at what is called  $N$ -extended supersymmetry.
  - B. One of the more remarkable things about this construction is that the equations of ordinary physics can be rewritten over superspace.
  - C. However, there is an upper limit to superspace.
    - 1. Many lectures ago, we introduced the fact that the force of gravity is somehow connected with the curvature of space and time.
    - 2. When this idea of Einstein's is applied to superspace, a limit is found. The highest number of copies of the anti-commuting coordinates that can be introduced is eight!
- IV. Ordinary particles moving in ordinary space and time don't change their identities. In the mathematics of superspace, however, if an ordinary electron moves into one of these extra directions, the electron becomes one of the selectrons!
- A. This gives us a simple understanding of why superpartners appear, and it is similar to Feynman's earlier observation that one way to understand the presence of antiparticles is to think of particles moving backward in time. In the notion of superspace, when an ordinary particle moves into one of the superdirections, it becomes a superpartner.
  - B. This means that supersymmetry is just a statement that ordinary particles are free to move in any direction in superspace. In fact, in the mathematics of superspace, it is straightforward to show that the superpartners obey exactly the equations of our world.
- V. There is one funny aspect of moving around in superspace.
- A. Imagine some children playing with a ball on a soccer field. One child starts in one corner of the field and kicks the ball; the ball rolls along the boundary of the soccer field until it reaches the next corner and a child there stops it. That child kicks the ball in the orthogonal direction, and it then travels along the boundary until it is stopped at the next corner. The children continue this game until the ball has traveled around the perimeter of the soccer field.

When the ball gets back to the first child who kicked it, it's still a ball.

- B.** Imagine now that these children have access to superspace. The first child kicks the ball in one of the directions of superspace, but when it reaches the second child, it's no longer a ball; it's a superball. When the second child kicks the superball to the third child, it changes back to an ordinary ball. Ultimately, when the ball reaches the first child again, it has changed four times, from ordinary ball to superball, back to ordinary ball, back to superball, and finally, back to ordinary ball.
- C.** One might think that the first child wouldn't notice any difference in kicking the ball in a complete circuit in ordinary space directions vs. superspace. However, the equations of superspace indicate that when the ball returns to the first child, it is hovering above the ground!
- D.** In this story, superspace is the soccer field, but the area above superspace is ordinary space. In moving around in superspace, whenever you try to return to the starting point after moving in superspace, the ball returns back into ordinary space in a different position. Mathematicians call this property *torsion*.

**VI.** Does Einstein's idea about curvature apply to superspace? Remember, in ordinary space, the notion of curvature of space and time leads to a description of gravity. If superspace is so much like our space, can it support curvature?

- A.** This question was answered in 1997. Wess and Zumino, and later Deser, found it is possible to write a curved supergeometry—a construct that incorporates Einstein's notion of curvature into superspace.
- B.** In ordinary space, incorporating curvature leads to gravity. In superspace, incorporating curvature leads to supergravity. Recall that the existence of the gravitino was important to explain why the superpartners are so much heavier than ordinary matter. Thus, application of the ordinary laws of physics to the strange mathematics of superspace generates a consistent view.
- C.** One can also apply the concept of superspace to Maxwell's theory, and the result is photons and photinos. In fact, every single equation of fundamental physics discovered for the past 150 years

can be described in the mathematics of superspace and results in the superpartners.

- D.** In Einstein's quest to explain gravity, he had to learn new mathematics to describe curvature. This same mathematics more or less exists for superspace. But the discovery of Einstein's equations in their most fundamental form in superspace presented a problem for a short while. Warren Siegel and I worked together at Harvard in 1977 to find the superspace analogs to Einstein's equations.

**VII.** The idea of superspace explains the fact that there are five strings.

- A.** An open string added together with 16 of these Grassmann coordinates that support vibrations moving in a left-handed sense is an *open type-I superstring*. Why do we add 16 Grassmann coordinates? The answer is that a spinor in 10 dimensions must have 16 components, not 4.
- B.** A closed string added to 16 Grassmann coordinates that support both left-moving and right-moving vibrations is a *closed type-I superstring*.
- C.** A closed superstring added to 32 Grassmann coordinates results in a *closed type-IIA superstring*. In this superstring, the 10-dimensional spinors are also both left-handed and right-handed and support left-moving and right-moving vibrations.
- D.** A closed superstring added to 32 Grassmann coordinates results in a *closed type-IIB superstring*. In this superstring, the 10-dimensional spinors are all left-handed and support left-moving and right-moving vibrations.
- E.** Finally, the closed string added to 16 Grassmann coordinates that move only in a left-handed sense plus the angles associated with gauge theories leads to the heterotic string.
- F.** No other consistent ways exist to play this game. Counting the number of formulations that come out of this theory leads precisely to five.
- G.** Let's review these five superstrings again:
- 1.** Open superstring + 16 left-spinning anti-commuting numbers (that support only left-moving vibrations) = open type-I superstring.

2. Closed superstring + 16 left-spinning anti-commuting variables (that support both left- and right-moving vibrations) = closed type- I superstring.
3. Closed superstring + 16 left-spinning anti-commuting variables (that support both left-moving and right-moving vibrations) + 16 right-spinning anti-commuting variables (that support both left-moving and right-moving vibrations) = closed type-IIA superstring.
4. Closed superstring + 32 left-spinning anti-commuting variables (that support both left-moving and right-moving vibrations) = closed type-IIB superstring.
5. Closed superstring + 16 left-spinning anti-commuting variables (that support only left-moving vibrations) + a number of right-moving vibrations that describe either  $SO(32)$  or  $E_8 \times E_8$  = heterotic string.

**VIII.** In the last lecture, we considered the fact that string theory is now being used to explain how quarks and gluons are held in the interior of nuclear matter. The ending point for the last lecture was AdS-CFT, also known as the *Maldecena conjecture*.

- A. Your lecturer's own work was undertaken in the context of a IIB string. Thus, this theory requires a IIB string and an anti-de Sitter geometry (i.e., a space with negative curvature). The output of this theory is not a theory of gravity, but a theory that contains the gauge fields that occur in the standard model. This shows how current string theory is attempting to make connections to the real world.
- B. In 1986, working with two of my students, Roger Brooks and Floyd Muhammad, we showed that the dimensions associated with forces in our world exactly follow the patterns that are appropriate for string theory. This is the work that permits the exact appearance of the Kemmer angles in string theory
- C. For anyone who has endeavored to follow this lecture, it can be said that *you* know SUSY; it is a strange construction that has hidden directions, but the mathematics of superspace indicates that for every particle seen, there is one superpartner...if our universe is supersymmetric.

**Readings:**

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

Kane, *Supersymmetry: Unraveling the Ultimate Laws of Nature*.

**Questions to Consider:**

1. Explain how the idea of superspace leads to the superpartners.
2. Describe the five types of superstrings.

## Lecture Twenty-Three

### Can I Have That Extra Dimension in the Window?

**Scope:** The last lecture offered an explanation of the assertion that there are five different strings. There's a problem with this assertion: Strings supposedly describe everything. If so, how can there be five different "everythings"? An answer to this question may be found in a construct that is not a string theory, *11-dimensional supergravity*. A look beyond 11-dimensional supergravity suggests it is part of a larger and even more mysterious construct, *M-theory*.

#### Outline

- I. The starting point of this discussion is a review of the idea of a *mode*.
  - A. An animation shows a closed string vibrating in such a way that three lobes seem to appear and disappear as the string vibrates. We can also find strings in which more or fewer lobes seem to appear and disappear. The number of lobes seen on the string is what we mean by a *mode of vibration*.
  - B. Every time the string vibrates in a different way, from our perspective, we see a different particle. Rather than talking about the entire string, then, it is possible to consider only its simplest mode of oscillation—the one in which only two lobes would appear in our picture.
  - C. If we restrict ourselves to this simplest way of vibrating, a full consideration of the entire string is no longer the focus; only a part of the string is the subject of this part of the lecture. This idea can be applied to the five strings that were known in 1994: the open type-I superstring, closed type-I superstring, closed type-IIA superstring, closed type-IIB superstring, and heterotic string.
    1. Truncation of the open string, and considering only its lowest mode of vibration, leads to 10-dimensional supersymmetric Yang-Mills theory.
    2. Truncation of the closed type-I string, and considering only its lowest mode of vibration, leads to 10-dimensional  $N = 1$  supergravity + super-Yang-Mills theory.

3. Truncation of the closed type-IIA string, and considering only its lowest modes of vibration, leads to 10-dimensional  $N = 2A$  supergravity.
4. Truncation of the closed type-IIB string, and considering only its lowest mode of vibration, leads to 10-dimensional  $N = 2B$  supergravity.
5. Finally, truncation of the heterotic string, and considering only its lowest mode of vibration, leads to 10-dimensional  $N = 1$  supergravity + super-Yang-Mills theory, but there are some slight differences here from the truncation of the closed type-I string.
6. In all these truncations, the world of strings has been left behind, and the equations investigated are more similar to the equations that describe the standard model.

**D.** Many of these truncations were discovered outside the realm of string theory. For example, the 10-dimensional string was constructed using techniques that mimic more closely those used to construct the supersymmetric standard model.

**II.** In a previous lecture, we discussed the degrees of freedom of the graviton and the fact that these degrees of freedom can change depending on the dimension in which the mathematics is described.

- A.** In our world, the graviton has 2 degrees of freedom. This is given by the formula:  $1/2$  of  $D(D - 3)$ .
- B.** For  $D = 11$ , this formula yields 44 degrees of freedom for the graviton. The purpose of this lecture is to discuss supergravity in the context of 11 dimensions, but a return to our 4-dimensional world for a moment is also useful.
- C.** In our world, photons come in left-spinning varieties and right-spinning varieties, and their rate of spin is twice that of an electron. The number of degrees of freedom of a photon in our world is 2, found with the formula:  $(D - 2)$ .
- D.** As we've said many times, the photon is a force carrier that acts between charged objects. In a hypothetical world of 11 dimensions, could there be something like the electromagnetic force that is typically carried by photons? The answer is yes, with some complications.

1. A look again at an animation of a point particle begins to show the way. As far as we have presently seen in the laboratory, things like electrons act like point particles.
  2. Previously, however, we noted that, in addition to point-like objects, we can also construct “lines,” called by different names, including *string* and *1-brane*. Consideration of, not a line, but a plane, leads to a *2-brane*. This can be replaced by a volume, which results in a *3-brane*. All of these objects can carry charge.
  3. Setting aside strings and just thinking about particles, how do branes couple to their force carriers? To answer this, we must introduce objects called *forms*. Forms are similar to photons, except they can couple to extended objects, whereas photons can couple only to particles. The photon is an example of a form.
- E. How many degrees of freedom does a form have?
1. We just discussed a sequence of objects: a point, a line, a plane, and a volume. The point was the 0-brane, and points couple to photons. The photon is also known as a 1-form.
  2. A single line is a 1-brane, and these couple to 2-forms, which are similar to photons. A plane is a 2-brane, and it couples to a 3-form.
  3. For every single object that can carry charge, there is a corresponding form that plays the role of the photon.
  4. Now let’s do some counting to find out the degrees of freedom for a form. For the 2-form, the formula for degrees of freedom is:  
 $\frac{1}{2}(D-2)(D-3)$ . For the 3-form, the formula is:  
 $\frac{1}{6}(D-2)(D-3)(D-4)$ . For  $D = 11$ , a 3-form has 84 degrees of freedom.
- F. Remember that a graviton had 44 degrees of freedom in 11 dimensions. Given that a 3-form and a graviton are bosonic particles, it is impossible to have supersymmetry with these objects alone. Supersymmetry requires both bosons and fermions. We can see, however, that  $44 + 84 = 128$ . Why is this number important?
1. Remember that, in superspace, gravitons can move into one of the superdirections and become gravitinos, the superpartners to the gravitons.

2. If there is a superpartner to the graviton in 11 dimensions, how many degrees of freedom does it have? This question was first answered in 1978 by three French physicists, Eugene Cremmer, Bernard Julia, and Joel Scherk. The answer is 128 degrees of freedom.
  3. The graviton together with the 3-form has 128 degrees of freedom. The gravitino also has 128 degrees of freedom, and supersymmetry requires a system to have a balance between degrees of freedom.
- G. This theory is called *11-dimensional supergravity*. It is not a string theory, but it uses the techniques of the standard model and Einstein's theory of general relativity in 11 dimensions.

III. Note that all the strings discussed exist in 10 dimensions, but the theory of supergravity exists in 11 dimensions. All the superstrings have truncation to supergravity and related non-string models. To what is 11-dimensional supergravity related as a truncation?

- A. This is a natural question to ask since every single 10-dimensional non-string theory can be obtained as a truncation of the vibrations of the full string. In the same way, physicists began to wonder: Is 11-dimensional supergravity the truncation of a bigger theory?
- B. Attempts to try to construct this larger theory were made in the 1980s. Objects called *membranes* were studied to see if they could be truncated and result in 11-dimensional supergravity. The problem with this program was that the modes of vibration of the membrane were infested with tachyons. It seemed, then, that 11-dimensional supergravity was not tied to a bigger theory.
- C. In 1995, Edward Witten proposed a surprising answer. He suggested that the theory of 11-dimensional supergravity was the truncation of something called *M-theory*. M-theory is truly mysterious even to theoretical physicists.
- D. The way that Witten reached his conclusion about M-theory is rather interesting.
  1. He started with the type-IIA string in 10 dimensions. Then, he tried to apply the properties of the quantum world to his calculations in this theory. For example, in the quantum world, to calculate a force, Feynman graphs are added to get the quantum corrections. Witten's prescription was to try to add all possible pictures that describe a force.

2. The calculations from this process reveal that the theory in 10 dimensions resembles 11-dimensional supergravity.
3. This was the first suggestion that there was a connection between 11-dimensional supergravity and something that is extended like a string. Witten's proposal set off the third string revolution.

**IV. M-theory has other implications in physics.**

- A. Another illustration in which these theories are represented by dots of light serves this purpose. The dots here represent the five types of strings discussed and one dot represents 11-dimensional supergravity. Witten was able to find connections between supergravity in 11 dimensions and the closed type-IIA string in 10 dimensions.
- B. In the early 1980s, in my own research, I proposed some relationships between supergravities called *dualities*. What Witten had found was also a duality.
- C. Very rapidly after Witten, connections were found between all the strings and 11-dimensional supergravity via quantum corrections. This partially addressed the question of whether positing many different strings as a theory of everything meant that our universe had many different "everything's." If all the strings are part of a single mathematical entity, then we would have one theory of everything.

**V. At this point, theoretical physicists still know very little about M-theory and are still looking for calculations to support its existence.**

- A. In 1997, the most precise calculation to date in support of M-theory was proposed by Banks, Fischler, Shenker, and Susskind. We can use an allegory to help us understand the support provided by their work.
- B. Imagine the most powerful microscope ever conceived. It can see well beyond what our current technology supports. This microscope might be turned to look at the string.
  1. With the increase of the magnification, we find that what we thought had seemed a single strand, in fact, looks more like a strand of pearls.
  2. This is exactly what happened with the mathematical investigations of Banks, Fischler, Shenker, and Susskind.

Their calculations showed that the string is made up of something that looks like 0-branes, or particles.

- C. The beads on the strand of pearls had appeared in physics earlier as *partons*. They had been discussed in the literature when physicists were struggling to understand the structure of protons and neutrons.
- D. Partons are calculationally useful; that is, there are experimental physicists to this day who use the parton idea to calculate the properties of nuclear matter. However, partons don't exist on the fundamental level; they were displaced by the notion of quarks.
- E. In the calculations of 1997 involving 10-dimensional Yang-Mills theories, partons were found. Furthermore, the calculations supported the idea that partons were the beads in the string of pearls that seemed to describe the string. In other words, under the string itself may lie an even more fundamental object, what is now called an *M-parton*.
- F. We seem to have traveled in a big circle. We began these lectures by saying that particles do not allow us to reconcile the laws of the quantum world with the laws of general relativity and gravity. We then saw that replacing particles with filaments offers a possible resolution to this problem. We now find that our filaments may actually be like strands of pearls and that the pearls themselves are particle-like structures.
- G. M-theory continues to be less than well understood. The 1997 calculations work only in a construct called the *infinite-momentum frame*. Thus, there is still some degree of controversy about whether M-theory is correct.

### Readings:

Greene, *The Elegant Universe* and *The Fabric of the Cosmos: Space, Time and the Texture of Reality*.

Kaku, *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*.

Kane, *Supersymmetry: Unraveling the Ultimate Laws of Nature*.

**Questions to Consider:**

1. What does 11-dimensional supergravity tell us about the degrees of freedom of the graviton and the gravitino?
2. Describe the circle we've traveled thus far in our exploration of string theory.

## Lecture Twenty-Four

### Is String Theory the Theory of Our Universe?

**Scope:** I hope that these lectures have shown you that string theory is embedded in the fields of science and mathematics as we have known them for hundreds of years. String theory weaves together an amazing story with strands contributed by both mathematicians and physicists, some of whom are long dead and some of whom are working today. We know that string theory encompasses a large collection of mathematical facts, but we have not yet seen evidence of it in the laboratory. We are still looking in string theory for the physical insight that will be the equivalent of the “Man in the Elevator” that led Einstein to the theory of general relativity. My own research today is aimed at finding the denizens of supersymmetry in the hopes of contributing a new tool in physics for the solution to this problem.

### Outline

- I. String theory is a collection of non-trivial, mutually enforcing mathematical facts, but we have not yet seen evidence of it in the laboratory.
  - A. For me, string theory is one of the most remarkable concepts I’ve seen appear in the realm of applied mathematics. The structure of superstring/M-theory touches on a number of subjects in mathematics: algebraic geometry, calculus, differential equations, differential geometry, Lie group theory, number theory, topology, and many more.
  - B. Amazingly, this construction, which started in physics, has started to “eat” mathematics. Before string theory, the last intense conversation between mathematics and physics took place in the 1930s, driven by the discovery of quantum physics.
  - C. Often, physicists “invent” mathematics they need to describe experiments. From a mathematician’s viewpoint, this method lacks rigor. Mathematicians often find themselves in the position of taking the suggestions of physicists and turning them into real mathematics.

- II.** One way to define the word *serendipity* is “the occurrence and development of events by chance in a happy or beneficial way.”
- A.** The word was coined by Horace Walpole and drawn from a story entitled “The Three Princes of Serendip.”
  - B.** Superstring/M-theory has an extremely high S.Q.—serendipity quotient. The mathematical accidents that must occur in order for string theory to be logically rigorous are incredibly large in number. In fact, many researchers call the theory “string magic.”
  - C.** In physics, when we see an enormous number of mathematical accidents, we believe we are looking at mathematics that is tied to the real world. An example of this is when Dirac tried to write equations to describe the electron in a way that was consistent with Einstein’s rules of special relativity. He succeeded, but he also found that his equations demanded the existence of an antiparticle for the electron.

**III.** Why did we push physics all the way to strings?

- A.** The idea of atoms seemed good enough at one point, but when we start probing the atom, we find that it contains elementary particles.
- B.** We’ve studied elementary particles for 60 or 70 years. Literally tens of thousands of pieces of data agree with the mathematical description of elementary particles, called the *standard model*. But none of the experiments we’ve performed with elementary particles takes into account gravity. If we try to write a theory combining quarks, leptons, force carriers, etc., with gravity, the result is mathematical nonsense.
- C.** After creating the theory of general relativity, Einstein spent 30 years trying to find its successor, which he called the *unified field theory*. Einstein was disturbed by the fact that his calculations could be made in such a way as to describe a universe in which only space and time existed, with no matter.
- D.** String theory is fundamentally different from general relativity in that the equations of string theory absolutely require matter.

**IV.** String theory is not a complete story, nor is M-theory.

- A.** In the quantum world, we cannot distinguish between particles and waves, and the way to describe a wave is with the mathematical

tools of a *field*. In these lectures, I never mentioned a string field because the field is the boundary of our knowledge in superstring/M-theory.

**B.** A field-theory description for the open string was discovered by Witten in 1984–1985. For closed strings and the heterotic string, no one knows how to write field-theory descriptions, which will be the true analogs of Einstein’s theory of general relativity. Why can’t we find them?

1. In 1905, Einstein had his miracle year, but in 1907, he noticed workers on a roof opposite his office building and had a thought: If one of the workers should fall, he would not feel his weight.
2. Einstein later described this as the “happiest thought of his life,” because it led to general relativity. General relativity is the genesis of the Big Bang.
3. Notice that this theory didn’t come through mathematics but through a deep understanding of one aspect of the universe.
4. In an animation of Einstein accelerated in an elevator, the acceleration feels like the force of gravity to him. If a ball were thrown into the elevator from the side, it would appear to be falling. Using this same reasoning with a beam of light, Einstein concluded that light must be bent by gravity.
5. This story of “The Man in the Elevator” can be used mathematically to derive Einstein’s equations of motion. But we have no such story for string theory. Even though we have a vast collection of mathematical facts, we do not have a point of physical insight that tells us why string theory must be correct.
6. In this way, we can think of string theory as “The Search for the Missing Man in the Elevator.”

**V.** Superstring/M-theory almost makes some predictions.

- A.** String theory tells us we should expect ordinary matter to have superpartners. But finding the superpartners will be only indirect evidence of string theory.
- B.** Looking at the running of the coupling constants in experiments we can perform in the next decade, we may be able to detect supersymmetry.

- C. We may also find a boost for superstring theory by studying the fluctuations in the temperature of the cosmic microwave background.
  - D. Whether or not we find superstring theory as a physical principle, it seems secure as a mathematical theory.
- VI. As an active researcher, I am aware of the problems presented by string theory.
- A. We need to find for string theory the equation that plays the role of Einstein's equation in general relativity. By solving other problems in string theory, we might make a contribution to solving this big problem. My current research is aimed in this direction.
  - B. Let's look at a picture of SUSY. The two blue areas in this picture represent universes that contain only bosons, on the one hand, and only fermions, on the other. For the past five years or so, I've been studying the mathematics of this picture.
  - C. The arrows that point from the blue area on the left to the blue area on the right and vice versa represent the supersymmetry transformations. What's in the blue regions? This brings us to a part of mathematics known as *representation theory*.
  - D. When we talked about spinors and vectors using the analogy of arrows and balls, those were representations. For rotational symmetries, we know every single representation that occurs. But we do not know all the representations for supersymmetry.
  - E. By studying the mathematical questions associated with these representations, I hope to answer a fundamental question: What's in the zoo of supersymmetry? What's the complete list of denizens that inhabit the blue regions in our picture?
  - F. What does this research have to do with string theory? We know that strings vibrate in all possible ways; thus, we expect that we will find, in string theory, all possible representations.
  - G. My colleagues and I haven't solved this problem, but we have found interesting connections to other fields of mathematics, such as K-theory. We have found pictures of the representations in these blue regions, suggesting that we are reaching a deeper level of understanding.
  - H. If we're successful in our efforts to find all the denizens, we will turn our focus back to the string and ask whether the equation that

plays the role of general relativity for string theory can describe all these possible ways of vibrating.

**VII.** Here are some of the denizens we've found in the world of supersymmetry.

- A.** Michael Fox and I, by studying the mathematics of these blue regions, remarkably found that there are simple pictures that contain the information that describes these representations.
- B.** We chose the name *Adinkra* for these new denizens, from the African language Ashanti, defined as a symbol carrying hidden meaning.
- C.** These Adinkras have a number of interesting properties; for example, they can be folded up, similar to proteins. At this point, we are simply classifying the number of representations we have found.
- D.** In closing, we see a picture of one of the most interesting beasts my colleagues and I have met so far. The complicated figure on the left is a picture of the force carriers and their superpartners. In other words, in the blue regions, we have been able to identify the representations that have exactly the properties of the gluons and gluinos, photons and photinos, and so on. The figure on the left is a picture of the quarks and squarks, electrons and selectrons, and so on.
- E.** We hope that this graphical approach will allow us to find all the inhabitants of the blue regions. Should we do so, physics will have a new tool with which to study the mysterious world of M-theory.

### **Readings:**

Walpole, *Horace Walpole and His World: Selected Passages from His Letters*.

### **Questions to Consider:**

1. What is the most significant problem facing string theory?
2. What are some of the discoveries that may give a boost to string theory?

## Timeline

- 1686 ..... Newton completes the *Principia Mathematica*, which includes his laws of motion and theory of gravity.
- 1783 ..... Michell reasons out the concept of a black hole.
- 1811 ..... Fourier develops a general method for describing vibrating systems.
- 1834 ..... Russell observes a solitonic wave while riding his horse near a canal in Edinburgh.
- 1844 ..... Grassmann develops the mathematics of *anti-commuting numbers*.
- 1846 ..... von Haidinger discovers that he can detect the polarization of light with his eyes; this effect is called *Haidinger's brush*.
- 1854 ..... Riemann shows that geometry done on a flat plane versus any curved surface is mathematically consistent and defines the *curvature tensor*.
- 1869 ..... Mendeleev designs the periodic table of the elements.
- 1873 ..... Maxwell publishes his treatise containing the four equations of electromagnetism and showing that they imply electromagnetic waves that propagate at the speed of light.
- 1874 ..... Stoney posits that the atom can be broken into parts.
- 1888 ..... Hertz shows experimentally that radio waves exist in physical reality.

1897 .....	Thomson conducts experiments leading to the discovery of the electron.
1900 .....	Planck postulates that energy of light is carried in discrete packets, called <i>quanta</i> .
1905 .....	Einstein's "miracle year," during which he publishes papers on special relativity, the photon concept with application to the photoelectric effect, and Brownian motion.
1913 .....	Beginning of the quantum revolution with the publication of Bohr's quantum theory of the atom; Cartan writes the mathematical description of spin.
1916 .....	Einstein publishes the general theory of relativity.
1918 .....	Noether develops her theorem relating symmetry to conservation laws.
1919 .....	Observations by Eddington confirm general relativity's predictions of the bending of light by the Sun's gravity.
1921 .....	Stern-Gerlach experiment reveals that all electrons are spinors.
1923 .....	de Broglie argues for wave-particle duality.
1925 .....	Pauli formulates the exclusion principle and a new quantum property of the electron.
1927 .....	Heisenberg formulates the uncertainty principle; Wigner introduces the concept of parity conservation.
1928 .....	Dirac writes equations for the electron and discovers its antiparticle, the positron.

1930 .....	Pauli posits the existence of the neutrino.
1932 .....	Chadwick discovers the neutron.
1934 .....	Fermi develops the theory of beta decay.
1938 .....	Kemmer proposes the idea of isotopic charge space, which requires only angles to completely specify location.
1948 .....	Gamow develops the Big Bang theory; Feynman, Schwinger, and Tomonaga produce the theory of quantum electrodynamics, successfully uniting special relativity with quantum mechanics.
1953 .....	Stückelberg and Petermann show that the coupling constant in the quantum world is a function of the energy at which it is observed.
1954 .....	Yang and Mills develop Yang-Mills theories, providing the mathematics to describe gluons, w-particles, and z-particles.
1957 .....	Bardeen, Cooper, and Shrieffer explain the workings of superconductors.
1962 .....	Tachyon named by Feinberg.
1964 .....	Quark model developed both by Gell-Mann and by Zweig; Brout, Englert, Guralnik, Hagen, Higgs, and Kibble develop mathematics of spontaneous symmetry breaking.
1965 .....	Cosmic microwave background observed by Penzias and Wilson.
1966 .....	Stanford Linear Accelerator Center (SLAC) starts operation; Berezin

- introduces the classical description of spin by anti-commuting Grassmann variables.
- 1967 ..... Electroweak unification proposed independently by Weinberg and Salam, based in part on contributions by Glashow; Penrose introduces the concept of twistors.
- 1968 ..... Development of the original version of string theory based on the bosonic string; Veneziano develops the dual resonance model, later recognized as the theoretical base of the string theory version of quantum gravity.
- 1969 ..... Deep Inelastic Scattering Experiment at SLAC reveals evidence of quarks inside protons and neutrons; Adler, Bell, and Jackiw identify the presence of anomalies in the quantum world.
- 1971 ..... Neveu, Ramond, and Schwarz present a variation of the original string theory composition that permits the inclusion of “notes” that have spin rates that are odd integers times the rate of spin of the electron, thus moving physics from the first generation of the string, the bosonic string, to the level of the spinning string.
- 1972 ..... Modern view of quarks developed by Gell-Mann, Fritsch, and Bardeen.
- 1973 ..... Bekenstein suggests that black holes should have a well-defined entropy.
- 1974 ..... Georgi, Quinn, and Weinberg show that the rates at which coupling constants change can be calculated if the number and types of denizens are known; Salam

and Strathdee propose the concept of superspace.

- 1974 ..... Georgi, Weinberg, and Quinn derive equations that describe the trajectory of *running coupling constants*.
- 1974–1975 ..... Hawking verifies Bekenstein’s suggestion and proposes that black holes radiate heat through quantum processes.
- 1975 ..... Two groups, Ferrara, Freedman, and van Nieuwenhuizen in one and Deser and Zumino in the other, posit supergravity.
- 1977 ..... Completion of the standard model with the discovery of the bottom/anti-bottom meson by Lederman; Giozzi, Scherk, and Olive eliminate the tachyon with their hypothesis of supersymmetry; Wess and Zumino and, later, Deser write a curved supergeometry, a construct that incorporates Einstein’s notion of curvature into superspace.
- 1978 ..... Cremmer, Julia, and Scherk propose a theory of supergravity in 11 dimensions.
- 1982–1984 ..... Green and Schwarz show that the  $SO(32)$  superstring is mathematically consistent in 10 dimensions, marking the beginning of the first string revolution.
- 1983 ..... Experimental verification of electro-weak unification.
- 1984 ..... Gross, Martinec, Rohm, and Harvey write equations describing the heterotic string; Kazakov shows, in the

- supersymmetrical model, the strong, weak, and electro-magnetic forces all unify at the same energy.
- 1984 ..... Friedan shows string theory is the first system where quantum theory is consistent with equations of general relativity.
- 1984–1985 ..... Witten writes a field-theory description for the open string.
- 1986 ..... Kawai, Lewellen, and Tye show that the complicated quantum force law for gravitons can be written in the form of simpler quantum force laws for gluons.
- 1988 ..... Gates and Siegel find the final formulation of the heterotic string, in which Kemmer variables appear. This offers the possibility of describing *four*-dimensional strings.
- 1995 ..... Witten proposes M-theory, a single construction that encompasses all of the 10-dimensional superstrings, heterotic strings, and 11-dimensional supergravity.
- 1996 ..... Super-Kamiokande neutrino detector begins operations in Japan.
- 1997 ..... Maldacena discovers the AdS/CFT correspondence, the unexpected relation between theories involving gluons and a theory of gravity in a space of one greater dimension, marking the beginning of the second string revolution and increasing interest in string theory as a possible “theory of everything”; Banks, Fischler, Shenker, and Susskind perform calculations

supporting the idea that the string is made up of 0-branes called *M-partons*.

- 1998 ..... Experiments at Super-Kamiokande reveal that neutrinos have mass. Cosmic expansion of the universe is found to be accelerating.
- 1999 ..... Randall and Sundrum propose the brane-world model of the universe.
- 2004 ..... Gates and Faux propose Adinkra as a method of organizing the representations of supersymmetry.

## Glossary

**AdS/CFT correspondence:** A set of mathematical prescriptions that suggests the possibility of calculating the properties of gluons and quarks by instead considering a corresponding theory involving only gravity in a mathematical description of a space with one more spatial dimension whose geometry is given by a negative constant value for its Riemann curvature tensor.

**anomalies:** A charge that may be conserved while ignoring quantum effects but may not be so conserved when they are included. The non-conservation is caused by anomalies.

**anticommutativity:** The property that implies that when two objects (these may be numbers, functions, etc.) are multiplied in one order, the result is the negative of the number that results from using the opposite order.

**antimatter:** Composites made of the antiparticles of those found in ordinary matter. If equal amounts of antimatter and matter are brought together, each is completely destroyed, to be replaced by an amount of energy that satisfies Einstein's famous equation,  $E = mc^2$ .

**baryonic matter:** Matter composed mostly of baryons. Protons and neutrons are the most familiar forms of baryonic matter. The distinction between baryonic and non-baryonic is important because the processes that synthesized baryonic matter are tightly constrained by the occurrence of the Big Bang.

**baryons:** Particles of matter that contain triplets of quarks. The term *baryon* comes from the Greek word *barys*, meaning "heavy."

**beta decay:** The process by which a neutron is transmuted via the weak interaction into a proton, electron, and anti-neutrino associated with the electron. This process lies behind the natural radioactivity of such substances as uranium.

**Big Bang:** A mathematical solution to the theory of general relativity that implies the universe emerged from an enormously dense and hot state about 13.7 billion years ago.

**Big Ear:** A microwave radio telescope that first detected the cosmic microwave background (CMB), then called *three-degree radiation*.

**binary stars:** Two stars orbiting about a common center. They often appear as a single image in a telescope. Sometimes called a *double star*.

**black hole:** A solution to Einstein's theory of general relativity with the property that nothing, via the laws of classical physics, can escape falling into the black hole's singularity after coming within the radius of the black hole's event horizon.

**brane:** Used in the context of present-day mathematical physics to describe the collection of elementary objects that includes points, line segments, planes, etc. A point is a *0-brane*. A line is a *1-brane*. A plane is a *2-brane*. Within a given space with dimension  $D$ , there is a maximum brane, a *D-brane*, that is also called a *space-filling brane*.

**Calabi-Yau manifold:** A type of compactification technique applied to models with extra dimensions. The extra dimensions are assembled into three complex numbers (using the usual factor of  $i$ ) and restrictions are put on them so that the surfaces they describe possess special properties.

**Chan-Paton factor:** Originally, the mathematical constructions that described the charges that could be appended to the ends of open strings. In modern interpretations, these mathematical objects describe the various ways that open strings can end on branes.

**charge conjugation:** The act of replacing an elementary particle by its antiparticle.

**charge conservation:** The fact that electric charge can neither be created nor destroyed. If the charge of an object changes, it must be because an electric current flowed to implement this change. This rule may be applied to other sorts of charges in place of electric charge.

**Clifford algebras:** The basic mathematical formulae, created in 1876 by William Clifford, that allow for a description of spinors.

**compactification:** The concept or process by which extra dimensions, considered actually to exist, may be subjected to additional mathematical restrictions so as to render the resulting models consistent with the physics of a space without extra dimensions. The simplest example is the projection of an image in our world onto a screen. Many mathematical properties of the object in our world must have a "shadow" description on the screen.

**computer-enabled conceptualization:** The concept that computers offer a genuinely new and alternative way for humans to access mathematics to

conceptualize properties and structures of the universe. Previously, this has mostly been enabled with the use of mathematical symbolic systems.

**concordance model:** The current view of the universe that posits that it possesses: (1) at the instant of the Big Bang, equal amounts of energy distributed between gravity and matter; (2) dark energy that explains its accelerating expansion; and (3) dark matter that controls galaxy lifetimes. Further, the model posits that the universe is composed of approximately five percent of the type of matter that is currently known to science.

**conformal system:** Any mathematical or physical system with the property that the physics it describes is the same when examined at all possible scales of sizes. This property sometimes is called *conformal symmetry*.

**cosmic microwave background (CMB):** A form of electromagnetic radiation (in the form of microwaves first found in 1964) that emanates throughout all directions in the universe. It can be regarded as the universe still reverberating from its Big Bang creation. As well, the CMB is a cosmic fossil giving concrete evidence for the occurrence of the Big Bang in the same way that ordinary fossils give concrete evidence for the existence of extinct life forms from Earth's history.

**cosmological constant:** A parameter or fudge factor originally introduced by Einstein into the equations that describe the properties of space and time. His original purpose for doing this was to describe a universe that is static and eternal. Today (using a sign opposite to the one chosen by Einstein), the cosmological constant is considered to be a possible source of dark energy.

**coupling constant:** A parameter in nature that measures how strongly a force affects objects that carry a corresponding charge. For example, the measured electrical charge of the electron determines how strongly the electromagnetic interaction affects the electron.

**coupling constant (running of):** Due to the effects of the quantum world, the value of a coupling constant changes depending on the energy at which the interaction occurs.

**covariant lattice:** An approach to constructing genuine four-dimensional strings via the use of bosons that describe crystal-like lattices. The "notes" produced by these, when combined with those from a four-dimensional bosonic string, are supersymmetric.

**CPT theorem:** The mathematical and experimental fact that combining the effects of charge conjugation (C), parity reversal (P), and time reversal (T) on the recorded data of an experiment involving elementary particles yields the same results as directly viewing the experiment.

**dark energy:** In astrophysics and cosmology, a form of energy that permeates all of space and explains the recently observed acceleration in the rate of expansion of the universe.

**dark matter:** Matter, not yet seen in laboratory experiments, that is distinct from ordinary matter and would explain the observed lifetimes of galaxies and the rates of rotation of stars in galaxies.

**Deep Inelastic Scattering (DIS) Experiments:** Name given to experiments that probed the interior of hadronic matter during the late 1960s to provide the first experimental confirmation that quarks exist inside hadronic matter.

**Delbruck scattering:** An experimentally verified process that demonstrates, due to quantum mechanical effects, that photons scatter off other photons.

**diffraction:** The ability of a wave to bend around corners. This phenomenon is easily seen with photons (or light), using an experimental apparatus to demonstrate regions of brightness and shadows. However, all waves demonstrate this property. It is the reason a person can be heard around a corner.

**DNA (Deoxyribonucleic acid):** A nucleic acid that contains the genetic instructions for all cellular life forms on Earth.

**dual resonance model:** The original name of string theory.

*e:* Transcendental number, similar to pi, with the property that it allows the simplest way to calculate logarithms. Its approximate value is 2.71828. The *e* stands for the first letter in the last name of Leonhard Euler.

$E_8$ : Refers to one of the *generalized rotation groups* found by the mathematician Cartan. He showed that all possible mathematical descriptions of rotation-like symmetries form four infinite families and six “exceptional” ones. The *E* in  $E_8$  stands for “exceptional.”

**Einstein’s hypotenuse:** A term created in this course to emphasize the important fact that Einstein’s special relativity theory can be viewed as a definition of a less well-known geometry—one distinct from that

commonly encountered in a non-technical education. For this course, this concept is a critical foundation used to gain more than a casual understanding of superstring/M-theory specifically and modern physics more generally.

**electromagnetic field:** The field of forces (associated with electric charges and magnets) that possesses electric components, magnetic components, and a definite quantity of electromagnetic energy.

**electromagnetic interaction:** One of the four forces in nature to which all electrically charged elementary particles are subject. The electromagnetic interaction possesses a single quantum known as the *photon*. When this force is studied only as acting on electrons, the results are summarized as quantum electrodynamics (QED).

**electron:** The most familiar of the subatomic particles in the lepton family and the first known elementary particle. It possesses a negative charge, as well as another kind of charge called *weak charge*. The electron is a stable particle.

**entropy:** A mathematical way to measure order or disorder in systems with statistical behavior. A state of maximum disorder possesses the lowest entropy.

**escape velocity:** The initial velocity required to send an object permanently away from a body possessing a gravitational force.

**event horizon:** A sphere that surrounds the singularity, which possesses several distinct properties. Nothing within the event horizon is ever able to escape eventually being “sucked” into the singularity. In principle, a light ray can orbit along the surface of the singularity. Outside the event horizon, the normal laws of gravitation apply. In particular, objects outside the event horizon can avoid being consumed by the black hole singularity.

**exchange force:** The force exerted on elementary particles that are fermions and only due to the fact that identical fermions must obey the Pauli exclusion principle. All other fundamental forces depend on the existence of conserved quantities, i.e., charge, energy, etc.

**exchanger:** A term introduced simply in the context of this course. More formally, in both mathematics and physics literature, this concept is called by the name *generator*.

**extra dimensions:** Described by the 18<sup>th</sup>-century scientist James Joseph Sylvester as “inconceivable,” this concept suggests that the universe may possess more than the three directions associated with length, breadth, and thickness. These are also called *hidden dimensions*.

**fermions:** Elementary particles that have a spin rate that is  $1/2$  times an odd integer times  $\hbar$ .

**Feynman diagram:** A graphical representation of the interaction between elementary particles and/or quanta of energy. Invented by Richard Feynman, these diagrams provide an extremely powerful tool for conceptualization and calculation.

**Feynman Rules:** Various elaborations of rules invented by Richard Feynman that allow the use of pictographic representations of quantum processes to be converted unambiguously into mathematical expressions. The expressions can then be compared with data from experimental observation.

**forms:** P-forms are mathematical quantities describing the analogs of photons; they couple, not to charged particles, but instead, to charged branes.

**free fermions:** This term is used in an approach to constructing genuine four-dimensional strings via the use of spinors that describe only supersymmetrical quantum theories in the presence of an anomaly. This construction is rather unique in that, here, the anomaly does not destroy the mathematical consistency of the constructions. These fermions are added to a four-dimensional bosonic string to provide a complete description.

**frequency (light):** Technically describes the rapidity with which either the electric or magnetic waves associated with a beam of light complete their oscillations when viewed from a point fixed in space. The frequency of a light beam is found by dividing the speed of light by the beam’s measured wavelength.

**fudge factor:** A value or parameter that is introduced into a mathematical formulation in an ad hoc way so as to produce a desired result.

**gauge invariance:** The property that, though two or more photons might be different, if the manner in which they change in space and time is the same, then the electromagnetic fields the distinct photons produce are identical.

More generally, this holds for any massless, force-carrying, spin-1 boson, and there are extensions to other values of spin.

**general relativity:** Albert Einstein's theory of gravitation developed by extension of the theory of special relativity to accommodate accelerating reference frames. It contains the introduction of the equivalence principle, which posits that gravity and acceleration are indistinguishable from each other. (This work is widely known as the theory of general relativity, or GR theory.)

**geometrical point particle:** The basic mental construct that enables all of classical physics as enunciated by Isaac Newton. The idea is to imagine an object of no measurable size, which nonetheless has the mass of ordinary objects.

**geometry:** The mathematical study of the rules and properties of figures, angles, and similar constructs.

**Glashow/Salam/Weinberg model:** The use of the mechanism involving the Goldstone particle to create appropriate masses for the W-plus, W-minus, and Z-zero quanta of the weak interaction. The mathematics of this demands the presence of a Higgs particle to generate mass for all other elementary particles.

**gluon(s):** These are to the strong interaction as the photon is to the electromagnetic interaction. These eight bosons are the quanta of this force.

**G minus 2 experiment:** A series of different experiments, almost continuously run for more than 40 years, designed to study whether an electrical property for elementary particles predicted by quantum theory is experimentally verified. This is the most accurately tested scientific observation ever made.

**Goldstino:** A fermionic analog of the Goldstone particle, whose role is to give mass to the gravitino, the spin-3/2 superpartner to the graviton.

**Goldstone particle:** A bosonic particle that potentially has the mass of a tachyon but avoids this fate by "rolling down a potential energy hill." In the process, this particle can cause force-carrying, spin-1 bosons to acquire mass.

**GPS (Global Positioning System):** A satellite navigation system, originally created by the U.S. military, to allow the accurate determination of position on the surface of the Earth.

**gravitation interaction:** One of the four forces in nature to which all elementary particles (including gravity's own quantum, the graviton) are subject. The word *gravity* is also used to describe this force.

**gravitino:** The superpartner to the graviton.

**graviton:** The force of gravity or gravitational interaction in the same way that the photon is the electromagnetic interaction. This is the quantum of the gravitational force.

**grey-body factors:** When light falls on an object in the real world, some is typically reflected and some is absorbed. A body that totally absorbs light is called a *black body*, and a body that totally reflects light is a *white body*. A *grey body* is one in which the light is mostly absorbed. Typically also, the amount of reflection versus absorption depends on the color (that is, the wavelength) of the light. Thus, a *grey-body factor* is a mathematical description of how the relative amount of reflection versus absorption depends on the color.

**hadrons:** Subatomic particles constructed as a composite of quarks, anti-quarks, and gluons.

**h-bar:** A physical constant of nature that plays a fundamental role in quantum theory. The  $h$  by itself is called *Planck's constant*, named for its discoverer, Max Planck, and  $\hbar$  is this number ( $h$ ) divided by  $2 \times \pi$ . Sometimes,  $\hbar$  is called the *Dirac constant* after the physicist Paul Dirac or, alternatively, the reduced Planck's constant.

**Higgsino:** A superpartner to the Higgs particle. Because Higgs particles are bosons, Higgsinos are fermions of spin rate  $1/2$ .

**Higgs particle(s):** In the standard model of elementary particles, the only known consistent description of the massive force carriers of the weak interaction (the Z-zero, W-plus, and W-minus bosons) requires the use of an elaborate procedure known as the *spontaneous symmetry-breaking mechanism*. In this mechanism, a Goldstone particle (or more than one Goldstone particle) "rolls" down an energy hill to allow the force carriers to gain mass. In all known realizations of this idea, there remains one spin-0 boson (or possibly more) with mass that allows all fermions to acquire mass.

**hypotenuse:** In a right triangle, refers to the side that is opposite to the right angle, that is, the angle of 90 degrees.

**intermediate vector bosons (IVB):** Collective name for the Z-zero, W-plus, and W-minus bosons. These are to the weak interaction as the photon is to the electromagnetic interaction. They are the quanta of this force.

**invariance:** The property of remaining unchanged by the action of a symmetry or some other definition involving change.

**K-theory:** A branch of the mathematics combining ideas from algebra and geometry that allows the study of whether the mathematical analogs of knots or tears occur on surfaces. Clifford algebras, which are related to spinors, play a role in K-theory.

**Large Hadron Collider (LHC):** Refers to a particle collider expected to be completed in 2007. The LHC is located in Geneva, Switzerland, and is to be operated at the European Center for Nuclear Research (known by its French acronym, CERN). The purpose of this device is to carry out research to understand the quantum world of nature in the greatest possible detail. If the Higgs boson or superpartners exist, they will most likely be discovered at LHC.

**Laser Interferometer Gravitational-Wave Observatory (LIGO):** A project designed to make the first direct experimental observation of waves of gravity, as predicted by general relativity theory of 1916.

**leptons:** A family of six elementary particles—distinct from gauge bosons and quarks—that each possesses a spin of  $1/2$ ; leptons do *not* experience the chromodynamic force but are subject to the electroweak force and the gravitational force.

**lightest supersymmetric particle (LSP):** In a universe where supersymmetry is valid, among denizens of the quantum world, the superpartners must occur with mass scales. Though these scales may be several hundred to a thousand times as massive as ordinary matter, one superpartner is the lightest. This is the LSP. In many models, the LSP is an excellent candidate for dark matter.

**loop quantum gravity (LQG):** A proposed alternative to string theory to create a consistent theory for quantum gravity. The starting point of LQG is the theory of general relativity without modifications. An important role is played by a mathematical quantity called a *plaquette*. As of this date, attempts to use LQG to reproduce the Bekenstein-Hawking formula describing the relation of entropy for a black hole have been problematic.

Even if this issue is resolved, unlike string theory, LQG has shown no signs of being a unified field theory.

**Lorentz transformation(s):** A set of equations introduced by Henrik Lorentz prior to the 1905 advent of relativity. Einstein adopted these to explain how measurements of space and time in one frame of reference are related to those in a second frame moving at a constant speed with respect to the first.

**metric:** A mathematical quantity that specifies how lengths, areas, volumes, and other measures are to be determined. This is the basic quantity that controls the rules of geometry. In physics, this quantity determines gravitational forces, along with the growth and evolution of the universe.

**mode(s):** In an extended system undergoing repeated oscillatory motion, overall recurring patterns may occur. These patterns may then be described as the *modes* of the vibration.

**M-parton:** The hypothetical point-like structures that may lie at the heart of M-theory. If M-theory is ever definitively demonstrated, it is likely that the strings of string theory will resemble a structure more like a string of pearls than a continuous pasta-like filament.

**M-theory:** An overarching mathematical structure within which all consistent string theories, as well as 11-dimensional supergravity, seem to emerge as special limits. No complete definitive proofs for the existence of this structure are known, but there are many pieces of circumstantial evidence.

**muon or mu particle:** One of the particles in the lepton family. It possesses all the same properties as the electron except it is 105 times as massive.

**neutrino(s):** Refers to any one of three particles in the lepton family. Neutrinos possess the same properties as the electron except they are electrically neutral and are thousands of times less massive.

**neutron:** One of the two most familiar of the subatomic particles in the baryon family. The neutron possesses a positive electrical charge and is 1,839 times more massive than the electron. The neutron decays typically after 1,013 seconds as a free particle. It is usually found in the nucleus of atoms.

**neutron star(s):** Star or stars of super-dense neutron matter that possess a powerful gravitational attraction, so that only neutrinos and x-rays can escape the interior of such stars.

**Newton's law of universal gravitation:** Mathematical result proposed by Isaac Newton to describe the gravitational force between planetary or stellar bodies.

**node(s):** In an extended system undergoing repeated oscillatory motion, points may occur that are at rest; these are described as the *nodes* of the vibration.

**Noether's theorem:** States that whenever there occurs a symmetry that may change in space and time (that is, a gauge symmetry), there must be an associated conserved charge accompanied by its own current.

**non-Euclidean geometry:** Refers to situations in which properties of figures, angles, and similar constructs are totally different from the (Euclidean) geometry typically taught in pre-college education.

**numeracy:** The analog to literacy in the manipulation of numerical and mathematical data.

**parity reversal:** The act of recording an experiment involving elementary particles by taking mirror-image data from the experiment.

**particle physics:** The branch of physics (also called high-energy physics) that studies the elementary particles associated with energy and matter and their interactions.

**particle zoo:** A term used colloquially to describe the extensive list of known elementary particles (they almost look like the list of animals in a large zoo).

**Pauli exclusion principle:** No two identical fermions can simultaneously possess the exact same quantum numbers.

**photino:** The superpartner to the photon. Because the photon is a boson, the photino is a fermion of spin rate  $1/2$ .

**photon:** The quanta of the electromagnetic force, also called a particle of light.

**$\pi$ :** The number obtained by dividing the circumference of a circle by its diameter. Its approximate value is 3.14159.

**Planck energy:** The amount of energy found by multiplying the fifth power of the speed of light by  $h$ -bar, next dividing this by Newton's constant, and finally, calculating the square root of this answer.

**Planck length:** The length that is found by first multiplying  $h$ -bar by the Newton universal gravitational constant, next dividing this result by the third power of the speed of light, and finally, taking the square root of the complete calculation.

**polarization (light):** The property of light that describes the pattern swept out by either the electric or magnetic waves associated with a light beam as it passes a single point fixed in space.

**polarization (circular):** If the polarization pattern swept out by the electric and magnetic waves associated with a light beam forms a circle, the light is *circularly polarized*.

**polarization (linear):** If the polarization pattern swept out by the electric and magnetic waves associated with a light beam forms a line, the light is *linearly polarized*.

**positron:** The antiparticle to the electron.

**power of 10:** A phrase used to indicate either a multiplication or division by a factor of 10. For example,  $100 = 10 \times 10$ , so 100 is 10 to the power of 2; dividing by 100 is 10 to the power of  $-2$ . Similarly,  $1000 = 10 \times 10 \times 10$ , so 1000 is 10 to the power of 3.

**Principia:** *Philosophiae Naturalis Principia Mathematica* (Latin for "mathematical principles of natural philosophy"), a three-volume work by Isaac Newton, published in 1687, that established the foundation of physics.

**proton:** One of the two most familiar of the subatomic particles in the baryon family. It possesses a positive electrical charge and is 1,836 times more massive than the electron. The proton is also a stable particle and is usually found in the nucleus of atoms.

**pulsar(s):** Rotating neutron stars possessing strong magnetic fields that can eject high-energy radiation from their north and south magnetic poles. The rotation causes the poles to point in constantly varying directions so that, when viewed from a fixed position, they seem to pulsate.

**Pythagorean Theorem:** One of the most famous results of geometry, as described by “the square of the hypotenuse of a right triangle is equal to the sum of the squares of its sides.”

**quantum, quanta:** The smallest discrete part(s) of energy or matter in the view of quantum theory.

**quantization:** This word has a number of meanings in physics. One definition, the one most commonly used in this course, refers to the fact that energy associated with any of the four fundamental forces comes in discrete packets, not in a continuum.

**quantum computer:** Theoretical type of computer in which the computations using bits (the basic unit of information in a computer) are performed by manipulating “entangled” bosons (such as the photon) or fermions instead of electrons, as is done in present-day computers.

**quantum entanglement:** Quantum theory suggests that objects that appear to be distinct in everyday observation are, in fact, associated with a type of wave, which contains information on all possible measurable attributes of the objects. The waves associated with distinct objects, under some circumstances, can be made to profoundly overlap. When this occurs, the information about the distinct objects becomes “entangled” so that a subsequent measurement of a property of one object contains information about the other(s). Photons are examples in which it is possible to investigate scientifically this phenomenon.

**quantum numbers:** The mathematical description of all the completely observable properties of an elementary particle.

**quantum theory:** One of the two foundational theories of 20<sup>th</sup>-century physics, postulating that matter and energy are capable of manifesting behaviors similar to particles or waves, depending on the type of observation to which they are subjected.

**quantum tunneling:** The process by which an object, typically an elementary particle, is able to travel through a region with a large energy barrier even though the object does not possess enough energy to accomplish this using the laws of classical physics (everyday experience). In the everyday world, a ball must be thrown upward very fast to travel very high. Via quantum tunneling, even if an electron were “thrown” upward very slowly, it would still be capable of reaching a very great height.

**quarks:** A family of 18 elementary particles, each possessing a spin rate of  $1/2$  and subject to the effects of the chromodynamic force, the electroweak force, and the gravitational force. To date, quarks have been observed only in hadronic matter, occurring in triplets (baryons) or matter-antimatter pairs (mesons).

**Regge trajectories:** Graphical plots in which two mutually orthogonal axes are labeled by the magnitude of the spin rate and the square mass. Tulio Regge noticed that, in many reactions involving the production of new hadrons, a subset of the new hadrons possessed properties remarkably similar to the ones that were present before the collisions. When the mass and spin rate of this subset are placed on such a plot, they fall along straight lines, which are called *Regge trajectories*.

**Riemann curvature tensor:** A mathematical quantity, related to the metric, constructed such that its values will be the same for two observers whose different descriptions of geometrical properties are due only to their different choices of reference frame.

**scalar:** Real and mathematical objects that possess the following property: After an observer walks around them for any distance along a circular path, they are found to look the same as they did on the initial viewing.

**selectron:** The superpartner to the electron. Because the electron is a fermion, the selectron is a boson of spin rate 0.

**singularity:** A geometrical point possessing no size at all that nonetheless has a mass that ranges from about five times that of the Sun up to virtually infinity. A singularity is the essential heart of a black hole.

**soliton(s):** Stands for “solitary wave” and is applied to their mathematical description and the actual waves seen in nature. Solitons many consist of a few non-repeating waves, and their mathematical descriptions have the properties that these waves “bounce” off one another like physical objects.

**special relativity:** One of the two foundational theories of 20<sup>th</sup>-century physics, this theory suggests that the speed of light is measured to be the same for all observers in motion without acceleration and consistently relates the measurements performed by such observers.

**speed of light:** The speed at which light travels in a vacuum, accurately represented by 299,792,458 meters per second. According to Einstein’s

work, no object or process containing information can travel faster than this rate, and only massless objects can achieve it.

**spin:** The intrinsic angular momentum of an elementary particle, which is the property of appearing to have attributes of tiny spinning balls, even though they possess no size at all. The rates of spin of elementary particles are measured in terms of a quantity called  $\hbar$  and come in any integer or half-integer multiplied times this rate.

**spinning string:** A string theory that incorporates the mathematics used to describe the spin of elementary particles. Its vibratory modes do not automatically incorporate equal numbers of bosonic and fermionic “notes” but can be made to do so with a certain truncation.

**spinning string (N = 1):** A string that incorporates the mathematics of spin as described for elementary particles. The key point of the construction is to introduce one additional anti-commuting strand for each real strand of the string. However, the anti-commuting strands behave as vectors when viewed before and after a complete circuit is made about them.

**spinning string (N = 2):** A string that incorporates the mathematics of spin as described for elementary particles. The key point of the construction is to introduce one additional anti-commuting complex strand (that is, using the square root of  $-1$ ) for each complex strand of the string. Once again, the anti-commuting strands behave as vectors when viewed before and after a complete circuit is made about them.

**spinor:** Real and mathematical objects similar to arrows (or vectors) that possess the following property: After an observer walks around them in a complete circle any *odd* number of times, they are found to point in the *opposite* direction as that seen in the initial viewing. After an observer walks around them in a complete circle any *even* number of times, they are found to point in the *same* direction as that seen in the initial viewing.

**spontaneous symmetry breaking:** When the Goldstone particle is at the “top of the energy hill,” all directions down the hill are equivalent. This is the meaning of a *symmetry*. After the Goldstone particle rolls down the hill, the direction it takes becomes special and the symmetry is broken.

**squark:** A superpartner to any of the quarks. Because quarks are fermions, squarks are bosons of spin rate 0.

**Stanford Linear Accelerator Center (SLAC):** A national laboratory in the United States operated by Stanford University. The 1.9-mile-long (3-kilometer) straight accelerator is underground and was the site of the DIS experiments that established the existence of quarks.

**Stern-Gerlach experiment:** The 1921 experiment conducted by Otto Stern and Walter Gerlach showing that electrons behave as spinors.

**string theory:** A mathematical theory proposed for describing all of physical reality; it suggests that modes of vibration of almost infinitesimal strings correspond to measurable fundamental properties of energy, matter, space, and time.

**strong interaction:** One of the four forces in nature to which only quarks are directly subject (it indirectly acts on all hadrons because quarks occur in the interior of hadronic matter). The strong force possesses eight quanta known as *gluons*. This force is sometimes also called *quantum chromodynamics* (QCD).

**Superconducting Super Collider (SSC):** A particle accelerator designed (but never constructed) to investigate the quantum world of nature and determine which of its possible mathematical descriptions actually occurs. Such a facility would be required to determine whether nature contains one or more Higgs particles, superpartners, and so on, or if some totally unexpected feature is present.

**supergravity:** An extension of the theory of general relativity in which at least one fermionic superpartner of spin rate  $3/2$  is consistently included in the mathematics of general relativity theory.

**Super-Kamiokande (or Super-K):** A neutrino physics laboratory designed primarily to study flux of neutrinos from the Sun, from supernovae, and in the atmosphere. Super-K was the site where the discovery of neutrino masses was confirmed.

**superparticle:** A mathematical description of both a particle of ordinary matter and/or energy simultaneous with its superpartner.

**superpartner:** In most systems possessing supersymmetry, for each boson, there must be a fermion that can be continuously “mixed” with it. The bosons and fermions allowed to mix this way are *superpartners* to one another.

**superspace:** A mathematical construct in which some directions are described by ordinary numbers with the usual commuting multiplication laws and some directions are described by Grassmann numbers with anti-commuting multiplication laws.

**superstring:** Superstring theory is a shorthand term for *supersymmetrical string theory* because, unlike bosonic string theory, it is a version of string theory that automatically incorporates equal numbers of bosonic and fermionic “notes” among its vibrations. The key point of this construction is to introduce additional anti-commuting strands that are multiples of the number 16 in addition to the 10 strands of a bosonic string. The anti-commuting strands behave as spinors when viewed before and after a complete circuit is made around them.

**superstring (heterotic):** A superstring that is constructed by adding to the usual bosonic strands of the string 10 additional Grassmann strands with properties corresponding to a vector and an additional right-mover to describe symmetries related to the gauge sector. This string can only be closed.

**superstring (type-I):** A superstring that is constructed by adding to the usual bosonic strands of the string 16 additional Grassmann strands with properties corresponding to one left-handed spinor with regard to the 10 dimensions of the string. The string can be either open or closed.

**superstring (type-IIA):** A superstring that is constructed by adding to the usual bosonic strands of the string 32 additional Grassmann strands with the properties corresponding to one left-handed spinor and one right-handed spinor with regard to the 10 dimensions of the string. The string can only be closed.

**superstring (type-IIB):** A superstring that is constructed by adding to the usual bosonic strands of the string 32 additional Grassmann strands with the properties corresponding to two left-handed spinors with regard to the 10 dimensions of the string. The string can only be closed.

**supersymmetry:** A mathematical symmetry between bosons and fermions in which there is no way to determine how much energy in a system is due solely to the bosons separately or solely to the fermions separately.

**surface of last scattering:** For a time after the Big Bang, the density of matter in the universe was so high that light could not freely travel through it. Recent results from the Relativistic Heavy Ion Collider (RHIC) suggest

that the universe may have been rather similar to a milky liquid during this phase. As the universe continued to expand, an instant occurred when photons, particles of light, were first freed to travel. In the instant before this, they had one last “bounce” from the “surface of last scattering,” a process rather like setting off a flashbulb. This “snapshot” of the infant universe can be seen today using microwaves (see **cosmic microwave background**).

**SUSY:** The abbreviation for “supersymmetry” or “supersymmetric system,” in which certain correspondences occur between bosons and fermions.

**symmetry:** An action in physics or mathematics that, when applied to some object, real or mathematical (a number, function, etc.), has the effect that before and after implementation of the action, the object looks the same.

**symmetry (continuous):** A symmetry that can be applied only in a continuous manner. As an example, imagine a round poker chip laying on a table and a game with two players. The game begins with the two in the room looking at the chip. Player A is sent out of the room, and player B may choose to rotate the chip on the table through any size of angle. When player A returns to the room, there is no way to decide through how much angle the chip was turned or whether it was turned at all while player A was outside the room.

**symmetry (discrete):** A symmetry that can be applied only in discrete “jumps.” As an example, imagine a poker chip laying on a table and a game with two players. The game begins with the two in the room looking at the chip. Player A is sent out of the room, and player B may choose to flip over the chip or not. When player A returns to the room, there is no way to decide which action took place while he or she was outside the room.

**Tachyon:** A type of never-observed elementary particle that, of necessity, travels no less than the speed of light. Tachyons also gain energy as their velocity is lowered, precisely the opposite of ordinary matter or elementary particles.

**taon or tau particle:** One of the particles in the lepton family. It possesses all the same properties as the electron, except it is 1,700 times as massive.

**time-like particle(s):** A particle that possesses negative energy due only to its motion and appears typically when the gauge invariances associated with massless, force-carrying, spin-1 bosons are disturbed.

**time reversal:** The act of recording an experiment involving elementary particles by first filming a movie of it, then taking data by viewing the movie while it is run in reverse.

**twistors:** The analogues of spinors for spaces in which there is more than one direction that appears in Einstein's hypotenuse with the same sign as the usual "time" direction.

**unified field theory:** A mathematical construct that contains all the elements to describe energy, matter, space, and time and gives an accurate and consistent view of our universe. Einstein (who labored unsuccessfully at its construction) suggested this name. Currently, this is also called by the immodest and misleading name, a *theory of everything*.

**vacuum:** The state of lowest energy in a physical system. In quantum theory, the vacuum is not the state of emptiness. Instead, via quantum processes, antiparticle/particle pairs spontaneously come into existence in the vacuum and annihilate each other.

**vacuum polarization:** A quantum process whereby a force-carrying, spin-1 boson has part of its energy converted into any number of antiparticle/particle pairs that eventually annihilate each other, thus returning all their energy back to the original boson.

**vector:** Real and mathematical objects similar to arrows that possess the following property: After an observer walks around them in a complete circle any number of times, they are found to point in the same direction as that seen in the initial viewing.

**wave function:** In quantum theory, a mathematical function used to describe the propagation of the wavelike properties associated with any particle or group of particles.

**wavelength (light):** Technically describes the crest-to-crest distance of either the electric or magnetic waves associated with a light beam. To the human eye, this is associated with the color of the light.

**weak interaction:** One of the four forces in nature to which all elementary particles (with the exception of the graviton) are subject. It possesses three quanta known as the *W-plus*, *W-minus*, and *Z-zero* bosons. This force is sometimes also called *quantum flavor dynamics* (QFD).

**Yang-Mills equations:** Equations developed by C. N. Yang and R. Mills that describe the physics of gluons and IVBs. These equations are the analogs of those developed by Maxwell.

**zero-point energy:** A classical oscillator, such as the pendulum of a clock, can hang undisturbed at rest. The laws of quantum theory forbid a pendulum in the quantum world from doing this. Because a quantum pendulum must always possess some motion, it must always have an associated energy. The smallest value of this is the zero-point energy.

## Biographical Notes

**Banks, Thomas** (born 1947): American physicist who, in 1999, in collaboration with Fischler, Shenker, and Susskind, gave the strongest evidence to date for the existence of M-theory at a fundamental level.

**Bardeen, John** (1908–1987): American physicist and the only person to be awarded the Nobel Prize in Physics...twice—first with Shockley and Brattain, in 1956, for the development of the transistor and then with Cooper and Shrieffer, in 1972, for achieving an understanding of superconductivity.

**Bekenstein, Jacob** (born 1947): Professor of theoretical physics at the Hebrew University of Jerusalem who contributed significantly to the science of black-hole thermodynamics.

**Bell, John** (1928–1990): Irish mathematical physicist who contributed to the deeper understanding of the foundation of quantum theory through his work known today as *Bell's inequality*.

**Bohr, Niels** (1885–1962): Danish physicist; with Einstein, the leading architect of the “old” (1905–1925) quantum theory of matter and energy and the leading interpreter of quantum mechanics in its formative years (1925–1935).

**Cartan, Élie** (1869–1951): French mathematician who studied rotations, among other topics, and wrote the first mathematical description of spinors.

**Clifford, William** (1845–1879): English mathematician who studied non-Euclidian geometry and developed what came to be known as *Clifford algebra*. His ideas played a fundamental role in Einstein's theory of general relativity.

**Cremmer, Eugene** (born 1942): Working with Julia and Scherk in 1978, he constructed the 11-dimensional supergravity theory that has led to the determined efforts to unravel M-theory.

**De Broglie, Louis** (1892–1987): French physicist whose 1923 doctoral dissertation extended early quantum theory by arguing that, just as waves behave like particles, particles must behave like waves. His prediction was confirmed three years later.

**Deser, Stanley** (born 1927): Professor of physics at Brandeis University; his research involves the study of quantum fields and gravitational theories.

**Dirac, Paul** (1902–1984): British electrical-engineer-turned-physicist, creator of the relativistic theory of the electron, and the founder of what became quantum electrodynamics. His prediction of the positron was a startling instance of the fertility of abstract mathematics for revealing physical reality.

**Eddington, Arthur** (1882–1944): Possibly the most important astrophysicist of the early 20<sup>th</sup> century, he headed joint expeditions to Sobral, Brazil, and the island of Principe to verify a prediction of Einstein’s gravitational theory.

**Einstein, Albert** (1879–1955): German physicist who made several fundamental contributions to science. In 1905 alone, Einstein demonstrated the atomic origin of Brownian motion, provided compelling evidence for the quantum theory of matter, and produced the first installment of his theory of relativity. In later life, Einstein struggled to find a grander “unified theory,” with no success.

**Feynman, Richard** (1918–1988): American physicist who, with Schwinger and Tomonaga, founded quantum electrodynamics. He invented the *Feynman diagram*, a simple sketch that allows one to think qualitatively about complex physical processes while retaining the mathematics “underneath” the pictures.

**Fourier, Jean** (1768–1830): French mathematician and physicist who is best known for a mathematical result known as the *Fourier series*, which provides an important tool to analyze the motion of any system undergoing repetitive motion.

**Gauss, Johann** (1777–1855): German mathematician who, in 1798, made one of the most important advances in geometry since the time of the Greeks; one of his discoveries is a foundation for the work of Maxwell.

**Gell-Mann, Murray** (born 1929): American physicist who formulated the *Eightfold Way*, organizing elementary particles into families and, with Zweig, created the quark theory of matter, later called *quantum chromodynamics*.

**Gerlach, Walther** (1889–1979): German physicist who, with Stern, conducted the Stern-Gerlach experiment, discovering the property of spin quantization.

**Goldstone, Jeffrey** (born 1933): American physicist who proved that, whenever symmetry is spontaneously broken, a particle called the Goldstone particle appears that can be used to give mass to spin-1 force carriers.

**Grassmann, Hermann** (1809–1877): German mathematician and linguist who invented a new algebra of vectors with applications in theoretical physics.

**Green, Michael** (born 1946): British physicist who, working with Schwarz, first formulated the superstring and later proved that all versions of the theory are free of anomalies.

**Gross, David** (born 1941): American physicist who, working with Harvey, Matrinez, and Rohm, invented the heterotic string in 1985.

**Harvey, Jeffrey** (born 1955): American physicist and co-discoverer of the heterotic string.

**Hawking, Stephen** (born 1942): British physicist, best known as a theoretician of space, time, gravity, and black holes. His work, along with that of Bekenstein, is fundamental to the search for a quantum theory of gravity.

**Heisenberg, Werner** (1901–1976): German physicist who developed the *uncertainty principle*, which states that one cannot, in principle, have precise simultaneous knowledge of the momentum and position of a particle.

**Hertz, Heinrich** (1857–1894): German physicist who, in 1888, showed that the prediction of Maxwell's equation regarding the transmission of electromagnetic signals through space was correct.

**Higgs, Peter W.** (born 1929): Scottish physicist and discoverer of the Higgs boson, a hypothetical elementary particle, strongly expected to be discovered in experiments to be undertaken between 2007–2020.

**Jackiw, Roman** (born 1939): Polish-born American physicist who, working with Bell, discovered the essential nature of anomalies in quantum theory.

**Julia, Bernard** (born 1952): With Cremmer and Scherk, he constructed the 11-dimensional supergravity theory, leading to efforts to unravel M-theory.

**Kaluza, Theodor** (1885–1954): Succeeded, in 1921, at the task that had eluded both Maxwell and Einstein by, with the introduction of one extra dimension, finding a unified description of electromagnetism and gravitation.

**Kemmer, Nicolas** (1911–1998): English physicist who, in 1938, offered the proposal that the similarities between the neutron and proton could be interpreted simply by considering that their charges are related to angles that exist in isotopic charge space. Unlike the idea of Kaluza and Klein, Kemmer's does not require the existence of extra dimensions.

**Klein, Oskar** (1894–1977): Swedish-American physicist who introduced compactification, required to consider that a mathematical universe with extra dimensions is not automatically ruled out by experimental observation.

**Lederman, Leon** (born 1922): Former director of Fermilab and professor of physics at the Illinois Institute of Technology, who shared the Nobel Prize with Schwartz and Steinberger for work with neutrino beams and has written *The God Particle* and other books aimed at increasing public understanding of science.

**Leibniz, Gottfried** (1646–1716): German philosopher and mathematician credited, independently of Newton, with the invention of calculus.

**Lobachevsky, Nikolai** (1792–1856): Russian mathematician who, in 1829, became the first person to publish a work on non-Euclidean geometry.

**Lorentz, Henrik** (1792–1856): Dutch physicist and 1902 Nobel Prize recipient for his work exploring the relation of magnetism and radiation. Although the conceptual framework of his derivation of Lorentz transformations relied on assuming the existence of the ether, Einstein adopted these and showed their consistency without requiring its existence.

**Maldecena, Juan** (born 1968): Italian physicist best known for his discovery of the AdS/CFT correspondence (also known as the *Maldecena conjecture*).

**Martinec, Emil**, (born 1958): American physicist and co-discoverer of heterotic string theory.

**Maxwell, James Clerk** (1831–1879): Scottish physicist who presented four equations that codify every aspect of electromagnetism, including the previously unrecognized phenomenon of electromagnetic radiation. Maxwell’s equations paved the way for the discovery of relativity and form the classical underpinnings of quantum electrodynamics, the quantum theory of light.

**Mendeleev, Dmitri** (1834–1907): Russian chemist who systematized the weights and chemical properties of 63 chemical elements in his periodic table of the elements.

**Michell, John** (1724–1793): British theologian, geologist, and astronomer who was the first person to consider the possibility of black holes and is given credit for proposing the existence of binary stars.

**Minkowski, Hermann** (1864–1909): Polish-German mathematician who, in 1907, realized that Einstein’s theory of special relativity has a simple interpretation in terms of space-time geometry (*Minkowski space*). Without this realization, it is doubtful Einstein would have been able to create his 1916 opus on general relativity.

**Neveu, Andre** (born 1942): French physicist who, with Schwarz, is credited with evolving the second generation of string theory (spinning strings).

**Newton, Isaac** (1642–1726): British natural philosopher and mathematician who developed calculus, the laws of motion, the law of universal gravitation, and principles of optics and light. Many of his ideas were summarized in the *Principia* of 1686.

**Noether, Emmy** (1883–1935): German-born mathematician who published, in 1918, *Noether’s theorem*, a work directly relevant to particle physicists.

**Pauli, Wolfgang** (1900–1958): Austrian physicist whose 1925 exclusion principle provided the first systematic explanation of the periodic table and opened the way to Schrödinger’s and Heisenberg’s versions of quantum mechanics. The Pauli exclusion principle states that no two quantum particles (of spin  $1/2$ ) may exist in the same quantum state. Pauli also predicted the existence of a new particle of nature, later called the *neutrino*.

**Penzias, Arno** (born 1933): German-born American physicist whose work (with Wilson) with a receiver called the *Big Ear*, monitoring radio

emissions for the Milky Way's encircling gas ring, led to the detection of the cosmic microwave background.

**Planck, Max** (1858–1947): German physicist who, in 1900, proposed the idea that energy comes in discrete bundles, called *quanta*, at the atomic scale. This theory helped to explain the spectrum of electromagnetic radiation emitted by a “black body” that absorbs all electromagnetic radiation that falls upon it.

**Ramond, Pierre** (born 1943): American physicist who independently discovered spinning string theories.

**Randall, Lisa** (born 1962): American physicist whose work with Sundrum posits that with the acceptance of the existence of at least one extra dimension and that our universe exists in a manner similar to a pane of glass, there is a simple reason to understand why the force of gravity is so much weaker than other fundamental forces.

**Riemann, Bernhard** (1826–1866): German mathematician whose contributions to geometry paved the way for Einstein's development of the theory of general relativity.

**Rohm, Ryan** (born 1969): American physicist and co-discoverer of the heterotic string.

**Russell, John** (1808-1882): Scottish engineer who first reported the observation of a solitary wave, which he called the *wave of translation*, and made one of the first observations of the Doppler shift of sound frequency.

**Salam, Abdus** (1926–1996): Born and raised in Pakistan, he made major contributions in the mathematical development of the standard model, in particular, the unification of the weak and electromagnetic forces of nature.

**Scherk, Joel** (1947–1979): French physicist who, with John Schwarz, was responsible for the realization that string theory possessed a mathematical description of the graviton and was, thus, capable of realizing Einstein's final dream of a unified field theory. Tragically, he committed suicide in 1979.

**Schrödinger, Erwin** (1887–1961): Austrian physicist; one of the giants of quantum mechanics. His wave mechanics of 1925 became the basis of Dirac's relativistic theory of the electron, which evolved into quantum electrodynamics.

**Schwarz, John** (born 1941): American physicist who, with Joel Scherk and independently of Pierre Ramond, invented the spinning string.

**Schwinger, Julian** (1918–1994): American physicist who shared a Nobel Prize in Physics for the complete theory of quantum electrodynamics, the quantum theory describing the interactions between electrons and photons.

**Siegel, Warren** (born 1951): American physicist who provided one of the major tools (called *chiral bosons*) necessary for the construction of the heterotic string, among many other fundamental contributions to field theory and supersymmetry. Dr. Siegel and I developed a third formulation of the heterotic string that is directly connected to the gauge transformations of the standard model and opened the way to a description of a four-dimensional version of heterotic string theory.

**Stern, Otto** (1888–1969): German physicist; together with Walther Gerlach, he conducted the Stern-Gerlach experiment, through which the property of spin quantization was discovered.

**Stoney, George** (1826–1911): British physicist credited with first proposing and later naming the first elementary particle, the electron.

**Sundrum, Raman** (born 1962): Indian-born American physicist who, in collaboration with Randall, proposed the *brane world scenario* (also called Randall-Sundrum models) as a model for the physical universe, in one of the most highly cited research papers of the era.

**Susskind, Leonard** (born 1940): American physicist who first realized that 18<sup>th</sup>-century mathematics relevant to elementary particle physics could be interpreted as arising from interactions of minute, filament-like objects subject to the laws of special relativity.

**t’Hooft, Gerardus** (born 1946): Dutch physicist and 1999 Nobel laureate who, in collaboration with Veltman, developed a practical way to calculate the effects of quantum theory of electromagnetic and weak interactions.

**Tomonaga, Sin-Itiro** (1906–1979): Japanese physicist who shared a Nobel Prize in Physics (with Feynman and Schwinger in 1965) for the complete theory of quantum electrodynamics, the quantum theory describing the interactions between electrons and photons.

**von Haidinger, Wilhelm** (1795–1871): Austrian geologist and physicist who discovered that he could visually detect the polarization of light. This effect is now called *Haidinger’s brush*.

**Wigner, Eugene** (1902–1995): Hungarian physicist and mathematician who received the Nobel Prize in Physics in 1963 “for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles.”

**Wilson, Robert** (born 1936): American physicist whose work led to the detection of what is now known as the cosmic microwave background.

**Witten, Edward** (born 1951): Mathematical physicist at the Institute for Advanced Study in Princeton who has made a number of contributions to the field of string theory and is the founder of M-theory.

**Zumino, Bruno** (born 1923): One of the leading experts in field theory and credited, working with Julius Wess, with constructing one of the first supersymmetric theories in four dimensions.

**Zweig, George** (born 1937): Originally trained as a particle physicist under Richard Feynman, he proposed the existence of quarks independently from Murray Gell-Mann. He later moved into the field of neurobiology.

## Bibliography

Abbott, Edwin. *Flatland: A Romance of Many Dimensions*. New York: Dover Publications, 1884. This work is well known among scientists and nonscientists alike as a fanciful story about a two-dimensional world that nonetheless displays real mathematical sophistication wrapped about a parody of life in 19<sup>th</sup>-century England, a rigid society stratified by class.

Ashton, Anthony. *Harmonograph: A Visual Guide to the Mathematics of Music*. New York: Walker & Co., 2003. Animations in this course were sometimes used to represent sound visually, which in turn, allowed us to use the similarities between the mathematics of strings and that of sound to construct useful analogies. An actual device from the 19<sup>th</sup> century directly constructed visual images from sound. This book explains the device and contains images of sound obtained by using it.

Bartusiak, Maria. *Einstein's Unfinished Symphony*. New York: Berkley Books, 2003. This book explores the context, history, and developments in the search to detect the waves of gravity predicted by Einstein's 1916 work. Such waves have never been detected, but Bartusiak presents a fun and informative introduction to the subject based on more than a decade spent following such developments.

Bernstein, Jeremy. *Einstein and the Frontiers of Physics*. Oxford: Oxford University Press, 1996. This book contains biographical information about Einstein, records of his interactions with other scientists, and discussions of the areas in physics that were paramount in his mind.

Burger, Edward B., and Michael Starbird. *The Heart of Mathematics*. Emeryville, CA: Key College Publishing, 2000. This book contains introductory information on many areas of mathematics, such as number theory, topology, knot theory, and geometry, that are critical to obtain a truly well-informed view of string theory.

Cajori, Florian. *A History of Mathematics*, 5<sup>th</sup> ed. New York: Chelsea Publishing Co., 1991. This book tells the story of progress in mathematics from the period of antiquity to the end of the First World War. Special attention is paid to the 19<sup>th</sup> and early 20<sup>th</sup> centuries.

Cheetham, Nicolas. *Universe: A Journey from Earth to the Edge of the Cosmos*. London: Smith-Davies Publishing, 2005. A beautiful collection of images highlights this book, illustrating many cosmological structures, including black holes, nebulae, planets, stars, supernovae, and white dwarfs.

Close, Frank, Michael Marten, and Christine Sutton. *The Particle Explosion*. Oxford: Oxford University Press, reprinted 1994. This book does for the microcosm what Nicholas Cheetham's does for cosmology. It is a brilliantly illustrated book that includes chapters with imaginative imagery of the denizens of the quantum world interspersed with imagery of the instrumentation and devices that allow humanity to study the microcosm.

Darrigol, O. "The Spirited Horse, the Engineer, and the Mathematician: Water Waves in Nineteenth-Century Hydrodynamics," in William Dunham, *The Mathematical Universe*. Hoboken, NJ: John Wiley & Sons, 1997. An essay on the career and interests of John Scott Russell, notable for his description of what he called the "Great Wave of Translation," whose mathematical description would prove critical to the discovery of the heterotic string a century later.

Gates, Jr., S. James, "On the Universality of Creativity in the Liberal Arts and in the Sciences," in *Beyond Two Cultures: The Sciences as Liberal Arts*. Santa Barbara, CA: Institute for the Liberal Arts at Westmont, 2006. ([http://www.westmont.edu/institute/pages/2005\\_program/panelists\\_2005.html](http://www.westmont.edu/institute/pages/2005_program/panelists_2005.html).) This essay contrasts fields in the sciences with those in the liberal arts and gives an extended description of my perception of the life and work of the theoretical physicist.

Gell-Mann, Murray. *The Quark and the Jaguar: Adventures in the Simple and the Complex*. New York: Henry Holt & Co., 1994. In this panoramic book, the Nobel Laureate physicist, who was also one of the scientists whose work was definitive in establishing the existence and description of quarks, sweeps through the breadth of fundamental theoretical physics.

Goldsmith, Donald. *Einstein's Greatest Blunder? The Cosmological Constant and Other Fudge Factors in the Physics of the Universe*. Cambridge, MA: Harvard University Press, 1995. In spite of its title, this book is actually a review of all of cosmology written in an accessible way. It touches on all the major issues in the field and gives explanations and an account of modern cosmology.

Greene, Brian. *The Elegant Universe*. New York: Vintage Books, 2000. The much-celebrated book that brought the topic of superstring/M-theory to the attention of the wider world and provided the basis for the NOVA/PBS television video program. The author shows a keen ability to guide his readers through a large part of the history and development of the subject.

———. *The Fabric of the Cosmos: Space, Time, and the Texture of Reality*, 1<sup>st</sup> ed. New York: Alfred Knopf, 2004. In this sequel to *The Elegant Universe*, Greene takes the reader deeply into the realm of questions about the very essence of space and time under the impact of discoveries in cosmology, particle physics, and superstring/M-theory.

Gribbin, John, and Mary Gribbin. *Annus Mirabilis: 1905, Albert Einstein, and the Theory of Relativity*. New York: Chamberlain Bros, 2005. In this work, the authors give a vivid description of Einstein's year of miracles, during which he announced to the physics community that a new "star" had appeared, although it would be another three years before this was apparent to the field.

Halpern, Paul. *The Great Beyond: Higher Dimensions, Parallel Universes and the Extraordinary Search for a Theory of Everything*. Hoboken, NJ: John Wiley & Sons, 2004. In this work, the author guides the reader through the maze of modern physics theory suggesting the universe may well possess extra dimensions that have not yet been detected through experimental instrumentation.

Harmon, P. M. *The Natural Philosophy of James Clerk Maxwell*. Cambridge: Cambridge University Press, 1982. All modern communication technology rests on Maxwell's work, which in this book, can be viewed through his scientific letters and papers. The work of Maxwell also provided the genesis for Einstein's miracle year with regard to special relativity.

Hawking, Stephen. *The Universe in a Nutshell*. New York: Bantam Books, 2001. In this work, the highly successful author-scientist carries the reader into the realms of black holes, dark matter, supergravity and supersymmetry, MACHOS, p-branes, proto-galaxies, and WIMPS. Also included are discussions of highly speculative prospects, such as time travel.

———. *A Brief History of Time*, 10<sup>th</sup> anniversary ed. New York: Bantam Books, 1998. This book marked an emergence, from the theoretical physics community into the popular realm, of the scientific discussion of the evolution of the universe. A challenging work that many regard as opening a new world for the public.

Hoddeson, Lillian, and Vicki Daitch. *True Genius: The Life and Science of John Bardeen*. Washington, DC: Joseph Henry Press, 2002. This book describes the life and work of the only individual in Nobel history to become the recipient of the Nobel Prize twice: the first for his contribution

to the discovery of transistors and the second for his work in solving a mystery, that of superconductivity.

Kaku, Michio. *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10<sup>th</sup> Dimension*. New York: Anchor Books, 1995. This work marked perhaps the first widespread public discussion of deliberations that had been occurring within the physics community for more than a decade. The author-scientist is a popular writer, as well as a media presence on many television scientific documentaries, and offers the reader insight into his perspective on these developments.

Kane, Gordon L. *Supersymmetry: Unveiling the Ultimate Laws of Nature*. New York: Perseus Books, 2001. This book marked the first discussion of the possibility that the universe may possess a totally unexpected property that implies the existence of yet-to-be seen forms of matter and energy known as *superpartners*. If they exist, these would mirror known forms of matter and energy. Many readers have found this to be a most challenging book that requires a great deal of effort in order to gain its benefit.

Kanigel, Robert. *The Man Who Knew Infinity: A Life of the Genius Ramanujan*. New York: Simon & Schuster, 1991. This book follows the arc of the life and career of one of the most unusual mathematicians in human history, Srinivasa Ramanujan. Like Einstein, Ramanujan started his rise to prominence as a clerk, but his unusual work and accomplishments in the area of mathematics called number theory took him from India to the halls of Cambridge University and a collaboration with the English mathematician G. H. Hardy.

Kaplan, Robert, and Ellen Kaplan. *The Art of the Infinite: The Pleasures of Mathematics*. Oxford: Oxford University, 2000. Historical figures important to the development of ideas that today go almost unnoticed are revealed, along with diagrams and figures, in an effort to render a tough subject readable for the nonexpert.

Krauss, Lawrence M. *Hiding in the Mirror: The Mysterious Allure of Extra Dimensions, from Plato to String Theory and Beyond*. New York: Viking Books, 2005. The author-physicist is well known for his popular works on the physics of *Star Trek*. In this work, he turns his attention to the topic of extra dimensions and, in particular, guides the reader to a substantial history on its evolution through several millennia of human history.

Lederman, Leon. *The God Particle*. New York: Dell Publishing, 1994. The Nobel Laureate physicist-author of this work initiated the first large-scale public discussion of the theory that all mass in the universe possesses a

single origin, the Higgs boson. Dr. Lederman provides a lively and accessible discussion in a voice that is unique among the physics community. The book also chronicles his career, which culminated in the recognition of the Nobel Prize for his work.

———, and C. T. Hill. *Symmetry and the Beautiful Universe*. Amherst, NY: Prometheus Books, 2004. In this work, Drs. Lederman and Hill undertake to open wide for public display the concept of symmetry. This concept has been central to the progress of fundamental physics since at least the time of Einstein. In fact, this was the primary guide that led him to his opus on general relativity.

Maor, Eli. *To Infinity and Beyond: A Cultural History of the Infinite*. Basel: Birkhäuser, 1987. One of the primary reasons behind the drive for a quantum theory of gravity is to avoid the presence of *infinities*. What are these? This fascinating book offers its readers insight into this question.

Miller, Arthur I. *Empire of the Stars: Obsession, Friendship, and Betrayal in the Quest for Black Holes*. Boston: Houghton Mifflin Co., 2005. Though Eddington was the scientist who announced to the world that Newton's view of gravity had been overthrown by observations that supported Einstein's, he was not receptive to the radical new ideas on black hole formation offered by Subrahmanyan Chandrasekhar. This book chronicles not just the conflict between these views but also the culture of the field during the early and mid-20<sup>th</sup> century.

Mlodinow, Leonard. *Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace*. New York: Free Press, 2002. Written by a young scientist who witnessed and was part of the period that brought superstring/M-theory to its current position in the field of physics, this book offers a necessarily personal view on a key member of the community that developed the theory. In addition, it opens a unique view for the reader on how geometry has evolved up to its current place in superstring/M-theory.

Motz, Lloyd and Jefferson Hane Weaver. *The Story of Mathematics*. New York: Plenum Press, 1993. The authors of this book, one of a pair of very similar works, give a nice overview similar to the concept behind these lectures, describing the intricate interweaving of ideas and demonstrating linkages to concepts from antiquity to the present in the evolution of mathematics.

———. *The Story of Physics*. New York: Avon Books, 1989. The authors of this book, one of a pair of very similar works, give a nice overview similar to the concept behind these lectures, describing the intricate

interweaving of ideas and demonstrating linkages to concepts from antiquity to the present in the evolution of physics. The interplay between mathematics and physics is a highlight.

Nahin, Paul J. *An Imaginary Tale: The Story of  $\sqrt{-1}$* . Princeton, NJ: Princeton University Press, 1998. For most people, the concept that some numbers are more “complex” than others might seem odd. In fact, the numbers that are formally called *complex numbers* have had a 1500-year history to reach the status of an accepted idea. This book tells this story and reveals that the creation of mathematics resembles other human creative endeavors in surprising ways.

Oerter, Robert. *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*. Upper Saddle River, NJ: Pi Press, 2005. Dr. Oerter’s work goes a long way toward pointing out for the public one of the towering scientific achievements of modern physics. The *standard model* is not speculative but one of the best tested pieces of science ever developed. This work lays out the intricacies of the scientific paradigm in a readable manner.

Penrose, Roger. *The Road to Reality: A Complete Guide to the Laws of the Universe*. New York: Alfred A. Knopf Publishing, 2006. One of the field’s most distinguished members lays out arguments that suggest alternatives to the conventional wisdom of the field in this well-received work. The agenda of this book is not just to elucidate unusual physics but also to point out curious connections to the human consciousness in the perception of reality.

Randall, Lisa. *Warped Passages: Unraveling the Mysteries of the Universe’s Hidden Dimensions*. New York: HarperCollins Publishers, 2005. This is a work by one of the most cited physicists in the world, who is also on the faculty at Harvard University. It has thus far shown all the signs of being a widely popular work, bringing physics to the public in an interesting and engaging way. The author presents the view held by a substantial part of the physics community that the extra dimensions of string theory are real. Perhaps an interesting twist is that it is not only superstring/M-theory that is simplified by this assumption.

Rucker, Rudy. *Infinity and the Mind: The Science and Philosophy of the Infinite*. Basel: Birkhäuser, 2004. This book, while providing commentary on the mathematical notion of infinity, also takes its readers to the realm of tantalizing philosophical questions.

Sagan, Carl. *Cosmos*. New York: Ballantine Books, reissued, 1997. Though dated and suffering from enormous shifts caused by new data, this book, as well as its accompanying NOVA/PBS video presentation, still sets the benchmark against which all such efforts must be measured.

Smolin, Lee. *Three Roads to Quantum Gravity*. New York: Basic Books, 2001. This book offers a lively debate about alternatives to superstring/M-theory. In the process, the author clearly enunciates the issues to be resolved by a mythical quantum theory of gravity. This book should provide some thought-provoking considerations for those asking whether Einstein's dream could be realized by some work other than superstring/M-theory.

Supplee, Curt. *Physics in the 20<sup>th</sup> Century*. New York: Harry N. Abrams, 1999. The highlight of this book is its visual offerings taken from diverse realms of the field of physics. It provides a balanced view of the various strains of physics from the 20<sup>th</sup> century. In particular, it covers background material to much of the discussion of the quantum world and the geometry of relativity in this course.

Susskind, Leonard. *The Cosmic Landscape: String Theory and the Illusion of Intelligent Design*. New York: Little, Brown, 2005. One of the fathers of the field of string theory tackles some of the thorniest current issues in the field. About 20 years ago, it was the predominant view that the solution to the equations of string theory would be unique and could describe our universe completely. Currently, a substantial number of physicists believe the opposite. Superstring/ M-theory may possess a large, perhaps infinite, number of consistent solutions that today are sometimes said to be elements of a "landscape." Implications of this idea for our existence are considered in this challenging read.

Tyson, Neil De Grasse, and Donald Goldsmith. *Origins: Fourteen Billion Years of Cosmic Evolution*. New York: W.W. Norton, 2004. If Sagan's *Cosmos* has a successor in tackling such an enormous wealth of issues in thinking about the cosmos and humanity's place in it, this book is as likely a contender for this title as any I know.

Walpole, Horace. *Horace Walpole and His World: Selected Passages from His Letters*. Boston: Elibron Classics, 2005. The word *serendipity*, in fact, had a beginning that was...serendipitous. This book chronicles some of the life and ideas of the man who brought this word to prominence in the English language.

Weinberg, Steven. *The First Three Minutes*. New York: Basic Books, 1993. Though this book is dated, it is still one of the finest expositions of what modern physics can say about the evolution of the universe near the time of its beginning. This work was probably the first to open for the public a presentation of what science can say on these issues.

Wells, H. G. *The Time Machine*. New York: New Review, 1985. The concept that time is the fourth dimension did not originate with Einstein. When he was 16, there was already a quite well known science fiction novel by H. G. Wells that possesses a surprisingly lucid description of this idea, which was already several millennia old. However, Einstein's 1905 work was the first to point out exactly what was wrong with Wells's ideas (and all who came before).

Wigner, Eugene. "On the Unreasonable Effectiveness of Mathematics in the Natural Sciences," in *Communications in Pure and Applied Mathematics*, 13(1). This essay clearly speaks to the unexpected magnitude of success that the mathematical paradigm set out for the field of physics by Galileo and Newton.

Zee, A. *Fearful Symmetry: The Search for Beauty in Modern Physics*. Princeton, NJ: Princeton University Press, 1999. This book is still one of the best available to the public to clarify the power of the concept of symmetry in the advance of fundamental physics since the time of Einstein.